553

Upper bound projection and Stochastic isovists

a solution to the comparison of Visibility Graph Analysis systems

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ABSTRACT

Visibility graph analysis (VGA) (Turner et al., 2001) is a widespread technique in the space syntax community, used to evaluate urban and building designs. To compare different spatial configurations, architects and researchers have relied on the use of the measure integration, based on D-value relativisation. This paper points out that this form of relativisation was developed originally for convex and axial analysis, the graphs of which have different structural properties compared to the kinds of networks found in typical VGA graphs. Using a new technique of incrementally increasing visibility graph density, we show that the integration value for a fixed point does not remain constant as the size of the graph/network increases. Using several graphs, we empirically demonstrate this non-consistency. We evaluate Tecklenburg (Teklenburg et al., 1993), NAIN (Hillier et al., 2012) and depth-decay methods of relativisation and show that these processes do not empirically produce the necessary stability required. From this, we conclude that it is difficult to compare integration values for VGA analyses using the traditional measure of integration based on known relativisation techniques.

To solve this, we introduce the notion of Restricted Random Visibility Graph Analysis or R-VGA. A fixed number of randomly distributed points spread over the system. Using R-VGA as a gold standard, we then introduce a new empirical relativisation called upper bound projection relativisation (UBPR) specifically for the relativisation of gridded, non-gridded and other dense isovist graphs. In conclusion, we suggest that traditional integration relativisation (the D-value) will always create only approximate solutions. Using UBPR, it is now possible for researchers to accurately compare different spatial models of different sizes.

KEYWORDS

VGA, Integration, Stochastic isovist, Relativisation, Space Syntax theory

1 INTRODUCTION

Visibility graph analysis (VGA) (Turner et al., 2001) is a widespread technique used in the space syntax community to evaluate urban and building designs. To compare different spatial configurations, architects and researchers have relied on the use of integration based on D-value relativisation. This paper points out that this relativisation was developed originally for convex and axial analysis, which have notably different structural properties to the kinds of networks found in typical VGA graphs.

For researchers and designers, it is often necessary to compare different buildings or designs, for example, comparing alternative design proposals for a particular urban or building intervention. Using VGA analysis, a rainbow colour spectrum is used to visualise isovist-generated, numeric data back onto the plan of the building, or urban system. For integration, this representation typically scales values from the most segregated value (blue) to the most integrated (red). When comparing spatial systems (buildings, neighbourhoods or cities) to a different spatial system or the same system with various intervention-options, it is often necessary to consider the individual integration values of representative points to detect changes in the overall distribution of integration. From the earliest space syntax literature (Hillier and Hanson, 1984), There has been a necessity to find ways of adjusting the values of total depth to allow for the size of the system (number of total nodes in the network). The earliest form was *d-value* adjusted integration introduced by Hillier and Hanson (ibid). D-value integration measures can be used in both axial and convex settings to compare different urban and building spatial systems. This can be extended to radius computations (considering only the local parts of the graph and not the entire graph). Given the large number of empirical findings reported based on these values, it seems safe to assume that they correctly perform the relativisation task they were designed for and we have no reason to suggest that integration calculations, using the d-value formula, are not appropriate for axial- and convex-based analyses. Therefore, when Turner first introduced VGA analysis (Turner et al., 2001), it seemed natural to apply the same integration equations to perform the same level of relativisation.

Recently, work has emerged which begins to question whether these kinds of relativisations are functioning correctly, in the case of VGA analysis. For example, (Zhang et al., 2013), when looking at the intelligibility (a separate measure that is based directly the measure of integration) of models, mazes and labyrinths, stated that their findings were that “*the measure of intelligibility which space syntax research claims is crucial for correlates to occur is inconsistent with particular spatial systems such as Toy Models*”. This suggests that something in the intelligibility computational process might be going wrong, in some situations, especially for the special case of VGA intelligibility. Returning to the measure integration, the first thing to be noted is that the equations of integration differ remarkably from normalised closeness centrality used in social network analysis (Bavelas, 1950). One reason for this might be attributed to the fact that there are different structural properties of Axial and Convex map-graphs compared to the graphs of social networks. The structural properties of VGA graphs are also noted as being different again, and indeed could be held to be far more similar to social network graphs. For example, in their book, an Introduction to Space Syntax in Urban Studies, van Nes and Yamu (van Nes and Yamu, 2021) mention the use of clustering coefficients citing (Watts and Strogatz, 1998) in relation to VGA but not on convex or axial networks. Again, this warrants a closer investigation into the functionality or appropriateness of the integration measure in the context of VGA analysis networks.

2 LITERATURE REVIEW

Hillier & Hanson first introduced the concept of relativisation in The Social Logic of Space (Hillier and Hanson, 1984). As mentioned, the objective of this mathematical process was to allow the comparison of different values of 'total depth' or 'status' in mathematics (Bavelas, 1950; Harary, 1994) between systems of various sizes. Total depth (T for space i, T_i) in space syntax is simply the sum of all shortest distances (d) from the origin node *i* to all other nodes *j* of consideration in the system of size n.

$$T_i = \sum_{j=1}^{n-1} d_{ij} \ j \neq i$$

Equation 1 Total depth for node, i, in a system

Clearly if the number of nodes in a graph changes, then the value for total depth will change. To allow for this, mean depth was introduced see equation 2.

$$MD_i = \frac{1}{n-1} T_i$$

Equation 2 Hiller and Hanson introduced Relative Asymmetry (RA) to produce a value between the minimum and a theoretic upper bound.

$$RA_i = \frac{2(MD_i - 1)}{n-2}$$

Equation 3 Relative Asymmetry for node RA_i

In practice, real buildings and axial-mapped urban systems tend not to have maximum total depths close to the theoretical maximum. To accommodate this real-world scenario, an upper bound, based on a hypothetical diamond shaped graph, was proposed to be the upper limit for maximum depth (Kruger, 1989). This creates the D-Value formula.

$$D_{n=}\frac{2\{\,n\left[log_2\left(\frac{n=1)}{3}\right)-1\right]+1}{(n-1)(n-2)}$$

Equation 4 D value formula for system of size n

The value used in space syntax analysis, including VGA. is typically that of Real Relative Asymmetry:

$$RRA_i=\frac{RA_i}{D_n}$$

Equation 5 Value of Real Relative Asymmetry RRA_i for node i in system of size n

It should be noted that the integration value is simply the reciprocal of RRA_i. The colour schemes used in both DepthMap (Turner, 2004) and prior software such as Axman (Dalton, 1997) tend to scale the colours from the highest (red) to lowest (blue) integration values available in the map rather than across the entire range of values from 1 to 0. Integration-based, step-depth changes has been used as a mainstay in most forms of space syntax research and is computed in all space syntax software. Next, we will consider some alternative forms of relativisations, less commonly used in space syntax analysis.

2.1 Teklenburg relativisation

Teklenburg, Timmermans and van Wagenberg (Teklenburg et al., 1993) critiqued Hillier's Integration method as being unstable and being unable to perform well in extreme cases. They also used the concept of a comparison with a hypothetic diamond-shaped graph as a basis for comparison. Through the use of a number of the numerical simulations, they suggested a new method shown in equation 6.

$$I = \ln\left(\frac{L-2}{2}\right)\Big/\ln({}_{\mathrm{T}}D-L+1)\,.$$ (L = N size of system)

Equation 6 Teklenburg revisitation for Node D for a system of size L

2.2 Normalised angular integration (NAIN)

With the introduction of angular measures (Dalton, 2001; Turner, 2001) and the consequential use of segmental analysis (Hillier and Iida, 2005) it became necessary to overcome the hitherto, traditional definition of 'distance' as being between nodes (either zero, an integer number of depth or infinite depth) and replace it with a real number (i.e. non-integer) value, typically between zero and one. Hillier, Yang and Turner (Hillier et al., 2012) went on to introduce

Normalised Angular Integration (NAIN) as a new way of normalising centrality for a segmental-based system. It should be pointed out here that angular, axial (i.e. un-segmented) systems still share a lot of the same fundamental graph characteristics with traditional axial and convex graphs. For example, segments have a relatively low node-degree and very low average values for their clustering coefficient.

$$NAIN_i = \frac{(N+2)^{1.2}}{T_i}$$

Equation 7 NAINi for node i in a system of N nodes. Note T_i is the total depth from node i but is typically not a whole value.

2.3 DEPTH DECAY

Finally, while not strictly a form of global relativisation, Dalton and Dalton (Conroy-Dalton and Dalton, 2007) introduced an alternative form of localisation. One of the many uses of integration is to allow for the different sizes of system when using a radius measure. Their ambition was to create a method that would allow for a richer way of highlighting local graph-structure but using a relatively simple process. Rather than ignoring values greater than a particular step depth, or radius, such as radius-three (R3), in depth-decay values in a graph are included, but further nodes add less to the total by employing a weighted contribution to total depth. They found that using a power of x where x=3.5 produces the effect of radius three integration (see equation 8):

$$DD_i = \frac{1}{(N-1)} \sum_{j=1}^{j=N} \frac{1}{(1+d_{ij})^x}$$

Equation 8 Depth decay for node I, where x is a parameter.

2.4 VGA

Turner introduced visibility graph analysis (Turner and Penn, 1999; Turner 2001), which is based on the fundamental spatial unit that is the isovist introduced by Benedikt (1979). Each isovist can be seen as a sampling of the available, visual space, in contrast to the other forms of spatial representation typically used in space syntax graphs, namely axial, segmental and convex spaces/analyses. Turner observed that isovists could, just as easily, be formed into a graph, as per previous space syntax analyses, by using the inter-visibility between any two isovist generating locations. If two isovist generating locations (the point from which an isovist is created) can mutually ‘see’ each other, then they form a connection, or edge, in the graph. Once the graph had been constructed, in this manner, then it becomes easy to compute centrality-like measures such as integration. This objective process eliminated the need for human operators to identify spaces by hand, since all navigable space could be automatically ‘flood-filled’ with an array of isovist generating points and this process could work directly from input data of buildings wall-surfaces, without a need for any additional manual sub-division. Turner demonstrated that isovist integration values reflected real-world observations in buildings (Turner and Penn, 1999). Since its introduction in 2001, VGAs availability in software such as Depth Map (Turner, 2004) visibility graphing analysis has become a mainstay of space syntax methods, especially for building-level analysis. As Field (van Nes and Yamu, 2021) mentioned, it is possible to combine the use of cluster coefficients with VGA; they mention how, *"Clustering coefficient is indicative of how much one loses in terms of visual information when moving from one location to another. Isovists that are closer to convex retain high clustering coefficient, hence little visual information is lost when moving from these locations"*. It should be noted that axial, segmental angular and/or convex analysis cannot use cluster coefficients as the value is close to zero and rarely changes for any meaningful values. This immediately suggests that the underlying structure of the graph differs between VGA and all other kinds of graphs encountered in space syntax.

Recently there have been some questions emerging over VGA. For example, Zhang Chiaradia and Zhuang (Zhang et al., 2013) suggests that intelligibility cannot be successfully applied to visibility graphs due to inconsistency and different values of N. Furthermore, Dalton et al. (2022) also observed that changes in rotation and the origin location of the VGA grid can lead to small changes in the average integration value.

3 DATASETS AND METHODS

The basis for assessing the applicability of the integration formula (the d-value formula) to VGA lies in the assertion that for a complete system, the integration value for a particular point in physical space must be insensitive to the density of the grid. Whilst adding more nodes to the system will, and should, increase the total depth of any system, the function of relativisation must be to eliminate the impact of the number of nodes in the system. From this, it can be stated that if we double the isovist point-density, we should *not* expect a commensurate change to the integration-value for the same point, i.e. the same location, on the plan or map.

The first empirical experiment, presented in this paper, was conceived to do precisely this: to choose a single point (i.e. location) on a plan and reprocess that plan using VGA of differing, increasingly greater, point-densities. As shown in Figures 1a and 1b, it is possible to process the same spatial information at different grid densities. It is also possible to examine this process numerically by selecting the point in space to determine what happens to its integration value. To do this, the nearest isovist, to a specified location, at each grid density can then be sampled and its integration value measured.

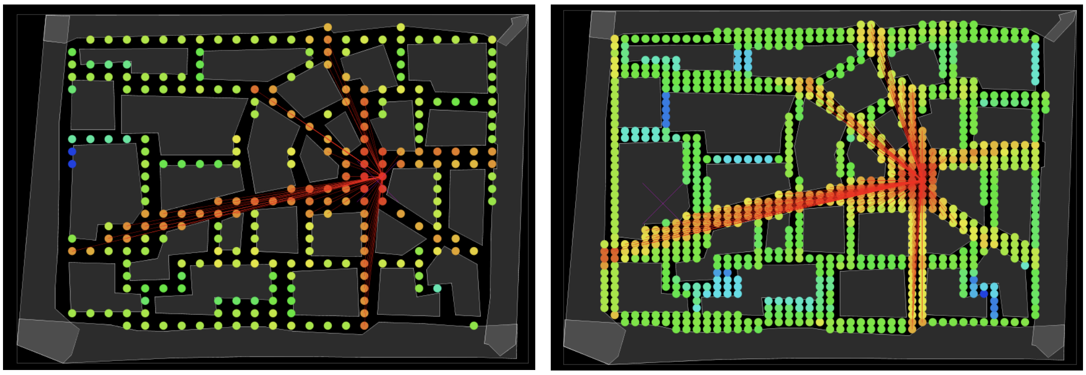

Figure 1: (a) Hillier's Intelligible world, an example of grid low density; (b) Hillier's Intelligible world, an example high gird density.

It is then possible to create a table of integration values for a particular, invariant, location plotted against the size, N (number of grid points), of the grid. It is clear that if integration were working correctly, then we would expect something approximating a horizontal, straight line, representing near-constant integration (some variance or noise would be acceptable due to fluctuations caused by digital representations of real numbers). From figure 2, we see something that, instead, approximates a reciprocal curve. This suggests that the relativisation process is far from insensitive to the size of the grid.

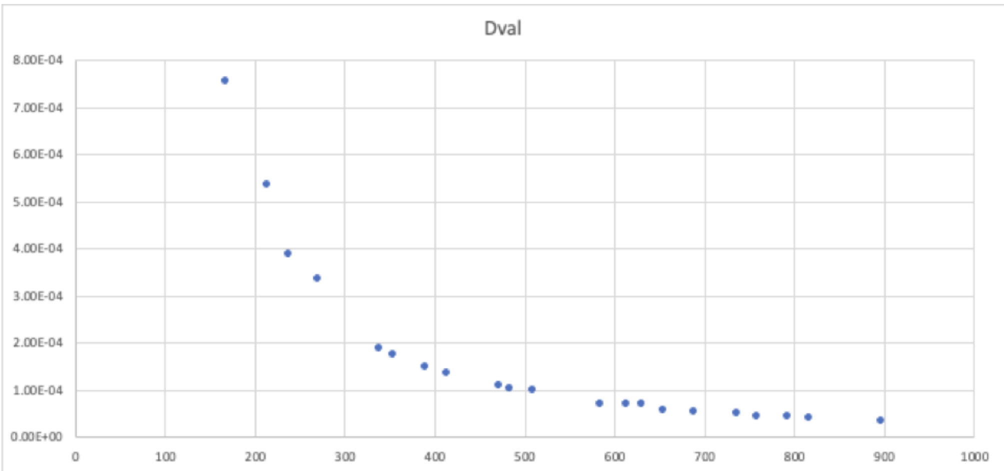

Figure 2 A plot of D-values for a single location in space, while changing the VGA point-grid densities.

One of the criticisms that could be made of this kind of experiment is that certain areas may be disproportionately over or under-represented as the grid density changes. For example, figure 1, shows a low-density grid which results in missing nodes near the bottom right-hand corner and down the left-hand edge. Another criticism is that the 'nearest node' (to the specified location) may shift slightly and so we would surely *expect* to see some small changes or fluctuations in integration as the location of the selected (nearest) point changes each time. For example, if the 'nearest isovist point' is adjacent to or crosses a threshold (for example, being either side of a doorway), we might certainly expect to observe shifts in its integration value, when conducting this type of experiment which would not necessarily be related to any failures, per se, in the relativisation process. However, to investigate this further, it becomes necessary to introduce a new kind of visibility graph analysis.

3.1 Restricted Randomised Visibility Graph Analysis (R-VGA)

Traditional visibility graph analysis begins by creating a regular, orthogonal grid (or raster grid) superimposed over the area being analysed (Turner et al., 2001). Each one of these points is then considered as a cell and is typically, but not necessarily about, a metre (or unspecified 'unit') wide. Grid cells (or raster points) are subsequently removed from the analysis if they are 'obstructed', for example fall inside a wall or inside any other solid or unreachable space/s. From this starting point, each grid then becomes the centre of a calculated isovist. Geometric properties of these resultant isovists can then be visualised using a heat map type representation, for example isovist properties such as area or area perimeter ratio (Benedikt, 1979) can be visualised in this way. It is also possible to create a graph representation of the relationship of one isovist to all other isovists: cells (or isovist generating points) by connecting if they are mutually visible to each other. This resultant 'graph' is what gives the visibility graph analysis its name.

This process of computing inter-grid or inter-isovist connections need not be limited to only using an orthogonal grid. It is equally possible to have any other type of grid, for example a hexagonal grid, a triangular grid or even one based on an Archimedes spiral (See Dalton et al, 2022 for a fuller discussion on this). From exploring alternative forms of grids, it rapidly becomes apparent that it is equally possible to use a stochastic process to place the isovist generation points, a process which was first proposed by Dalton (Dalton, 2011; see also Dalton et al., 2022) One possible objection to Dalton's stochastic isovist-point placement process is that by using random probability to create isovist generating points, or grid cells, it might be possible to create points that are very close to each other. So, for our definition of random visibility graph (**R-VGA**) that we are presenting we introduce a restriction to the point-placing process. After an initial seeding of points, new points are added incrementally. Every time a new isovist generating point, or grid cell, is added to the map, N, locations are considered (the value of N-10 was selected after considerable empirical testing). Each of these candidate cells is first checked so as not to be coincident with any solid/inaccessible zone, in a similar manner as for standard VGA analysis. The distance from each of the candidate points to their closest existing point, already in the system, is recorded. The candidate point from the consideration list with the greatest distance to its nearest neighbour is chosen and added to the system. This process is continued until a maximum number of pre-established points is achieved. This 'restriction' on an initially random process produces an evenly distributed system (see figure 3) and is known as Restricted Randomised Visibility Graphs, R-VGA (Dalton et al., 2022).

3.2 Computational tests using the R-VGA method

Now that the process of producing a *restricted randomised visibility graph*, or R-VGA, has been described, it is possible to return to the problem of producing a reliable empirical measure of the change of integration against grid density. As mentioned previously, it is quite possible to create VGA and R-VGA graphs of different densities. However, small changes in the size of grid cells for VGA analyses can lead to significant changes in the number of nodes in the system (Dalton et al., 2022). For our purposes, the major advantage of using a restricted randomised visibility graph, R-VGA, over a VGA graph, is that a new grid cell is introduced one at a time without affecting the location of the existing, already-placed, grid cells. This allows a very fine-grained tool with which to examine changes in any density-associated changes in the value of integration.

A second computational experiment can, therefore, be performed in the following manner. A plan of a building/area/site is initially seeded with 125 entirely random points. These 125 random points will be, more or less, evenly distributed and will form a skeleton, irregular grid or network. It is then possible to select a grid cell for investigation. In this experiment, four grid cells were selected, one close to the point of highest integration determined through traditional VGA analysis, one at a segregated location (determined as above), and two more added which occupy intermediate positions (semi-segregated and semi-integrated). A new, restricted randomised isovist point is then added to the system, the new inter-visibility analysis is computed producing a new graph (as per VGA), and the revised integration value for these four cells is re-computed. For each selected isovist, a plot can then be made of the integration value against the size of the system (N, the number of points or grid-cells). Once a note has been made of each of the four integration values along with the size of the system, another isovists cell is then added and the process repeated. Given that each cell is in a fixed position it is possible to analyse the precise changes in depth-values with no interference or noise arising from point-placement.

New software, created specifically for this purpose, was written in the Java language using the Processing IDE and library (Arnold et al., 2005). The code for the d-value calculations was transferred directly from the open-source C++ code for DepthMap (Turner and Varoudis, n.d.) from file PafMath.h into Java to avoid any transcription or recoding errors. Several environments were processed; an example test-environment is shown below, with its four selected points, this being Hillier's intelligible world (see figure 3) taken from (Hillier, 1996).

4 RESULTS

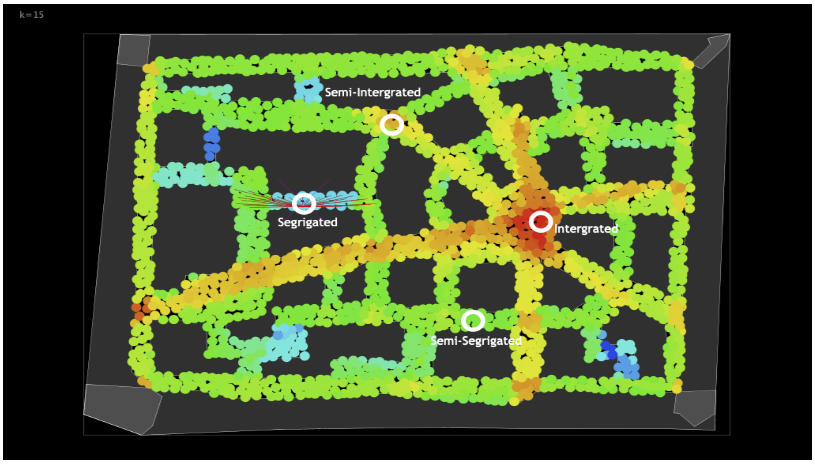

Figure 3 R-VGA Processing of Hillier Intelligible example with locations of grid-cell shown

4.1 Hillier and Hanson D-value based relativisation

In Figure 4, the vertical axis shows the integration values for each location, and the horizontal axis shows the size of the system (in this case, 125 to 2000 isovists). Firstly, this plot confirms the results of increasing the standard VGA grid density from Figure 2, except this shows more regularity and fewer gaps. It should be noted that the integration values are very low, the range being between 0.004730009 to 0.00000843805. Given that the purpose of most forms of relativisation is to normalise the values for easy comparison, the fact that these values are extremely low, possibly indicates that the integration value (based on the d-value formula) is not suited to this form of graph.

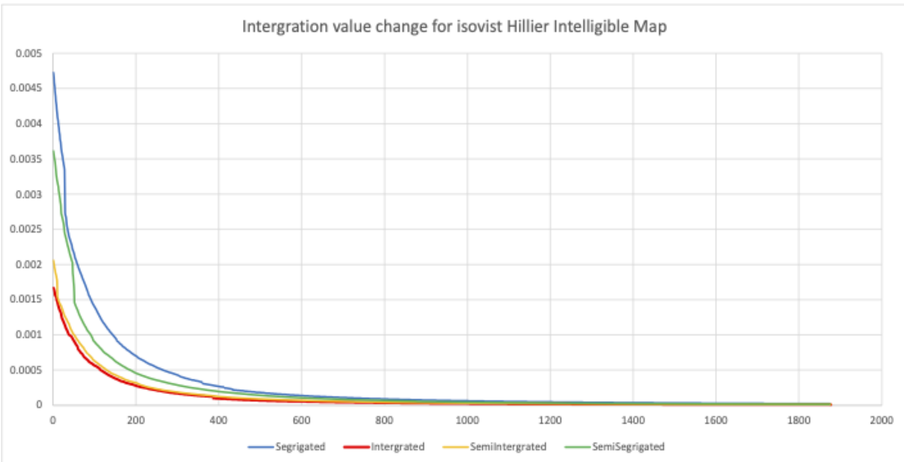

Figure 4 Plot of integration for the four chosen grid-cells.

Positively, this and other maps show that while the absolute value of integration changes, the rank order of the integration values, for the selected four points, never changes and the ratio between the values of the four is also practically constant. This can be measured by looking at the ratio of one integration value to that of the most integrated point, for a given value of the number of grid-cells, N. By looking across the ratios for all calculated values of N, it is equally possible to examine the standard deviation from the average values. This process shows that the ratio values change by, at the very most, only 6.5%. Given that many VGA maps are inspected visually, and the colours and distributions remain consistent (and it would be hard for the human eye to detect a 6.5% change in colour). This ratio consistency could also explain why the changes in integration have not previously been apparent to researchers working with VGA tools. While the differences in integration values remain invariant, the use of integration, and the d-value in most applications (particularly for non-VGA analyses) is unaffected. The critical use case comes when comparing different VGA maps with differing values of N.

Returning to Figure 4, the changes in integration value are generally very smooth, with the only significant change being caused by the addition of a grid cell isovist which results in a new 'visual shortcut' being formed in the graph, hence reducing the total depth of the system. For Hillier's intelligible world, the system became fully saturated after 400 grid cells (N=400) in this configuration. Once saturation is reached, we can reconfirm that while the relative values of integration remained constant, their absolute values changed. From this experiment, we can conclude that the relativisation process for integration using the d-value formula fails to remove the effects of the size of the system, unlike its performance for axial line graphs. For an insight into why this is not working, we can look at the value of K, the connectivity of an isovists grid cell: the most integrated grid isovist has a connectivity value ranging from K= 21 (N=125) to K=408 for (N=2000). These are far, far higher than found in a typical urban or building-level axial map. In other words, VGA isovist graphs are very different kinds of graphs.

4.2 Teklenburg relativisation

A similar process can be repeated for the other forms of relativisation introduced in the previous section. Again, to ensure consistency, the code for the computations was transferred into Java from the original DepthMap code (Turner and Varoudis, n.d.). Figure 5 shows a similar plot for the same 4 locations using the alternative form of relativisation proposed by Teklenburg Timmermans and van Wagenberg (Teklenburg et al., 1993). The original relativisation process was designed for axial maps and node graphs. As can be seen from figure 5, these values, too, are not stable across changes in the size of the system. From the chart, the reader might think that the values have more discontinuities than the integration case above. It should be noted that in Figure 5 the chart's axis begins from a value of 0.6, which exaggerates the differences, compared to the integration case.

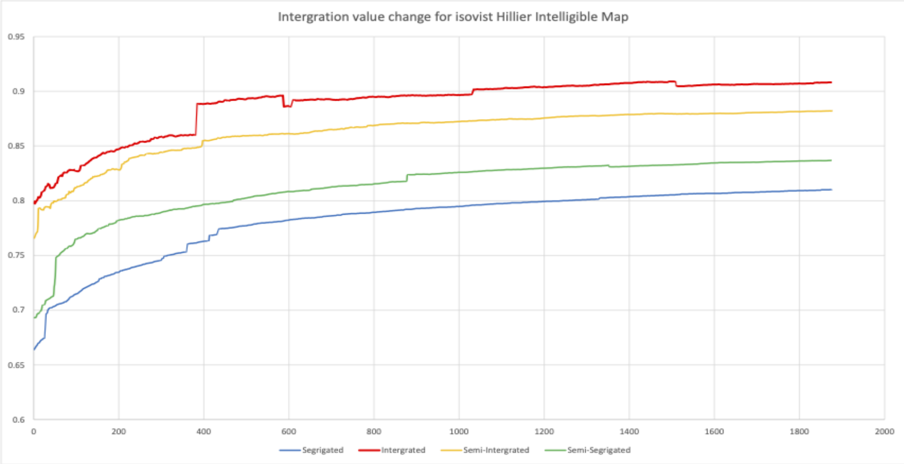

Figure 5 Plot of Teklenburg relativisation for the 4 chosen grid-cells.

Looking at figure 5 we can see that this excellent form of relativisation also has had some underlying assumptions broken in the case of VGA and R-VGA. This should not be seen as a criticism of Teklenburg et al.'s process as they were initially designed for entirely different types of graphs. However, from this figure, it would be apparent that comparing the absolute values of Tecklenburg integration for a system of size 400 isovists with one of 1800 isovists would lead to the false conclusion that the denser system is more integrated as a whole.

4.3 Normalised angular integration

Plotting values for Normalised angular integration (NAIN) permitted this form of relativisation to also be computed by our software; figure 6 below shows a plot for the same four locations.

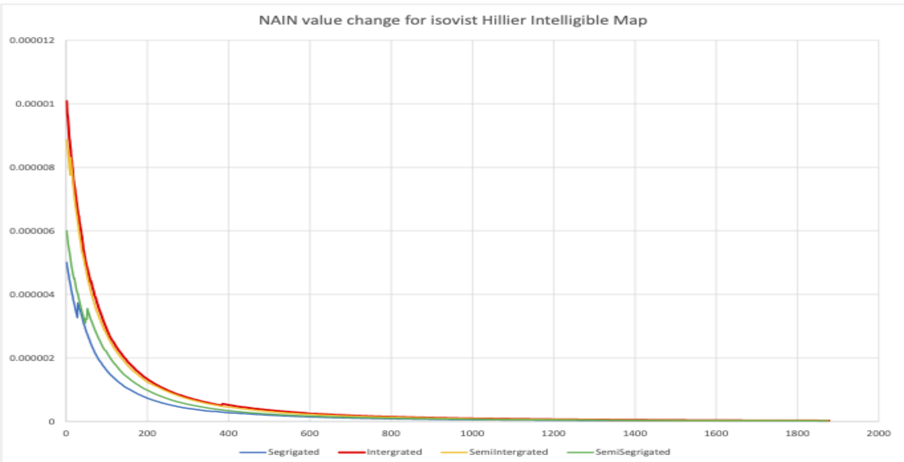

Figure 6 plot for NAIN relativisation for the four chosen grid-cells.

It can be seen from this plot, in figure 6, that the values are also not truly comparable against different sizes of the system. From this, it can be concluded that this is also not a valid way to relativise between different system scales for VGA and R-VGA type graphs. The underlying explanation of this is that again VGA and R-VGA break fundamental assumptions as to the structure of the graph, which are different from those of axial, convex or segmental maps. Visibility graphs are fundamentally different types of graphs.

4.4 Depth decay relativisation

Finally, we look at depth decay relativisation (Conroy-Dalton and Dalton, 2007). This was never intended to be another attempt to find a relativisation process but a way of generalising the process of calculating different radii (similar to the commonly used axial analysis use of radius). Depth decay typically tries to compare different regions within the same map and therefore, essentially, ignores the system's size. For the sake of completeness, it was determined worth investigating the use of the depth decay measure for the comparison of VGA and R-VGA.

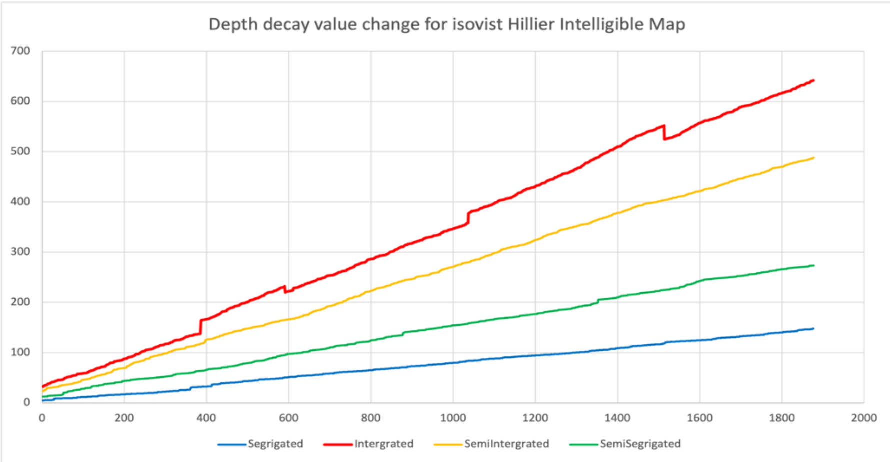

Figure 7 plot for Depth Decay relativisation for the four chosen grid-cells.

As is evident from Figure 7, Depth decay also does not permit integration values to remain invariant against different sizes of system, in the case of using RVGA (and so also VGA).

4.5 Proposed Relativisation Gold standard for VGA

As we have seen, none of the methods for relativisation presented above, appear to create numerically reliable results across different scales (isovist point numbers). This raises the question of how researchers could be enabled to compare different integration values across different sizes of spatial system. This paper would like to present two methods to solve this problem.

The first method is based on our new Restricted Randomised Visibility Graph Analysis (R-VGA). A principal difference between traditional VGA and the new R-VGA method, described above, is that it is easily possible to introduce a fixed number of grid cells, indeed, the total number of grid cells *must be specified* to provide the incremental placing of cells with a cut-off point.. For standard VGA it is challenging to adjust the grid density to reach a specific number of grid cells. A straightforward solution then is to choose a fixed number of grid cells for all systems. By using the same number of cells, the simplest form of 'total depth' can be used to compare the total depth values directly without any need for any sort of relativisation. Providing the fixed number is high enough to cover the system fully then it becomes possible to compare the total depths directly. This should be seen as creating a 'gold standard' by which other relativisation processes can be considered.

4.6 Proposed New Relativisation Method: Upper-bound Relativisation (UBR)

Finally, we introduce a new relativisation process, design specifically for VGA and R-VGA like graphs/networks. If a chart is plotted between the total depth in the system, on the y-axis, against the size of the system, on the x-axis (see Figure 8), it can be seen that the growth is generally linear against system size. From this perspective, one simple form of relativisation is to divide the total depth D by the size of the system (similar to the calculation for mean depth).

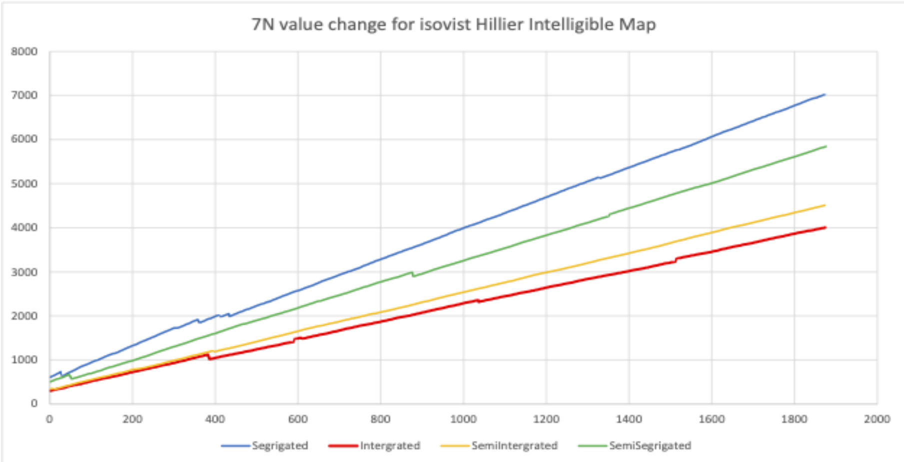

Figure 8 Total depth for 4 chosen grid-cells.

In practice, this produces numbers in the range of 1 to 6, so empirically we divide by (7 * N) to achieve a number closer to the 0.0 to 1 range.

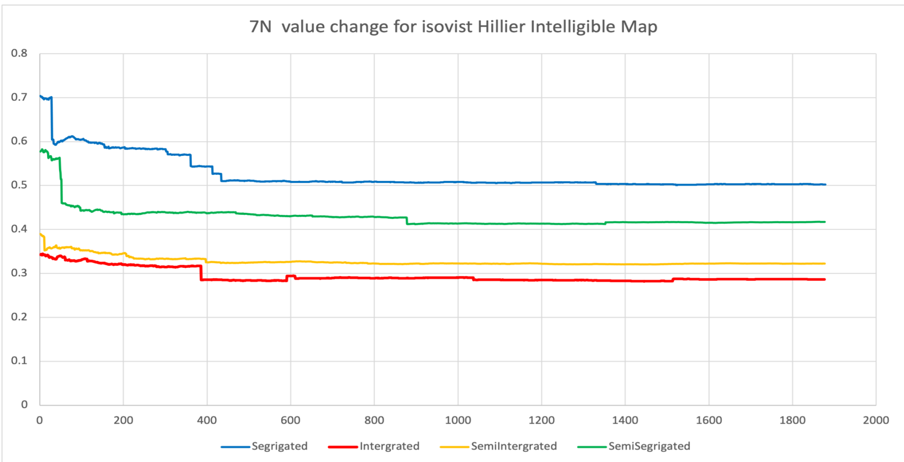

Figure 9 UBR relativisation for 4 chosen grid-cells.

Applying this, it can be seen, from Figure 9, that we can begin to observe the kinds of results we would expect from UBR Relativisation. Considering at the right-hand half of the chart, we see that the values are relatively static, indicating that the relativisation process is operating correctly. For systems bigger than 500 grid cells, the values are generally constant, as would be expected for a correctly relativised system. The standard deviation for all values of UBR, for systems greater than 500 isovists, tends, at most, to be 0.0239255, which is only a 1.64% deviation. Most of the variation for systems containing less than 500 isovists can be explained by the sudden, small changes in total depth which occur when the placement of a new isovist-point produced a new visual shortcut in the graph, a phenomenon we terming ‘connective leaps’ (see also our paper, specifically on ‘connective leaps’ and the Aha! Moment, McElhinney et al., 2022). Below 500 isovists, the total depth values do change, but it could be argued that this simply reflects the growing approximation to the ‘correct values’ and is noise caused by the low numbers of isovists. As such, we can confidently state that R-VGA should begin with a minimum of N=500 isovists, although larger values may be necessary for more complex system.

It should be noted that for UBR values range from 0.0 to 1.0, a very highly segregated location with a value greater than 1.0 could theoretically occur but is highly unlikely in everyday use.

5. DISCUSSION

From the computational experiments performed and presented in figures 3 to 9, it can be seen that the existing relativisation process (the d-value formula) does not allow a fair comparison between different sized systems when used with either VGA or R-VGA analyses. We are confident that the relativisation process would work, as expected, if a similar experiment were to be performed for axial maps or convex spaces using topological measures (for which the d-value formula was intended). It is not reasonable to assume that the previous, very general methods, that have been applied to a variety of different graphs for both comparison between various buildings and urban settlements and using both global and local radius measurements, would simply be perfectly transferable to visibility graphs. As shown in the case of VGA and R-VGA, the integration values do not appear to allow for the changes in size of system, in the same way as would be expected if the same experiment were performed for an axial map. It is proposed that this breakdown occurs because the graphs created by VGA and R-VGA have different inherent structural properties to those found in axial and convex situations. Specifically, VGA and R-VGA exhibit high levels of cluster coefficients which axial and convex maps do not.

Cluster coefficients measure the interconnectedness locally and is common in many social networks. For social networks, if node (person) A is linked to node (person) B and node (person) C, it is highly likely there is also connection between (persons) B and C. The cluster coefficients range between the value of zero, or no interconnections (neighbours are never connected at the first level), and one, every neighbour to node A is connected to every other neighbour. Axial maps and convex maps typically exhibit very low values of cluster coefficient (ranging from 0 to 0.1). The experiments presented here, working with dense VGA and R-VGA graphs produced cluster coefficient values close to 0.7 for Hillier's intelligible world and 0.78 for a shopping centre in Singapore (another world analysed but not presented here). It is our suggestion that this and other structural differences in the graphs negates the underlying assumptions inherent in previous work in this area (Hillier and Hanson, 1984; Hillier et al., 2012; Teklenburg et al., 1993).

When it is necessary to compare different systems, especially of a different size, this paper has proposed alternative mechanisms to achieve this. The first is to use R-VGA with a fixed number of isovists for both systems. If possible, this could be done with traditional VGA but, as mentioned, this is very difficult as the user has little control over this aspect of grid placement in Depthmap and trial and error would be the only mechanism by which this would be achievable. R-VGA simplifies the process, and it's recommended for that purpose. Equally, there are other advantages to R-VGA analysis, expanded upon in our other paper (Dalton et al., 2022). The second method proposed is that of UBR. This can be applied as a relatively simple form of relativisation, which would allow inter-system comparisons using the new, proposed UBR values.

6. CONCLUSIONS

Relativisation has been part of space syntax research since the earliest days of (Hillier and Hanson, 1984). It has a long and robust history of allowing the comparison between different types of axial and convex spaces. It has been used in the variety of contexts, becoming a familiar part of the fabric of space syntax itself. This paper has presented evidence that by examining systems at different densities of isovist grid, it appears the process of relativisation has not been working as originally envisaged, specifically for VGA and R-VGA graphs. This does not cast doubt on previous research, given that for systems larger than 2000 nodes/cells, the variation in integration values between 2000 and 3000 does not change substantially.

From the theoretical point of view, we do, however, question the absolute reliability of previous relativisation mechanisms when applied exclusively to VGA and R-VGA graphs. It should be noted that existing relativisation mechanisms became unreliable in cases for which they were never initially designed.

This paper has presented two alternative mechanisms for relativisation. The first is to use a restricted randomised visibility graph, R-VGA, approach using the same number of grid cells, or isovists, to analyse the system. Providing the value chosen is enough (more than 500 isovists) to create a dense model of all of the space, the values of total depth will be directly comparable. The second method, we call UBR, is a relativisation process tailored specifically for VGA and R-VGA systems. It should be noted that UBR does not claim to be a generalised relativisation process. It is highly and designed explicitly for VGA and R-VGA like systems. The UBR system is as attuned to the graph's structure as Integration or NAIN were to the specific form can character of axial and convex graphs.

The contribution to knowledge of this paper lies with the UBR as a method that can be used to directly compare the integration values between systems of different sizes. Additionally. The use of fixed-sized systems using R-VGA have also presented as an alternative solution and one that could be used to continue to investigate this sort of deeper level theoretical analyses. Were fixed sized R-VGA graphs to be used as a standard method, the space syntax community would have to agree on reporting certain sizes of N, for example N=1000 for small houses and buildings, N=5000 for larger buildings and N=10,000 for neighbourhoods, etc. Third, many researchers have been tempted to use VGA intelligibility but have steered away from it, since it is known to be problematic. It is possible that the issues with VGA intelligibility stem only from the failure of the integration formula, as applied to VGA graphs, and therefore fixed size R-VGA graphs or UBR applied to VGA could enable VGA/R-VGA intelligibility to be used with confidence. Finally, the methods developed in this paper for tracking the intelligibility value of a fixed point in space, over different versions and iterations of analysis also forms a unique contribution to space syntax's methodological knowledge-base, allowing future researchers methods by which we can validate alternative forms of relativisation for VGA-like systems.

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