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Doing Mathematics with Robots: an Activity Theoretical Perspective on the Links between Mathematics and Programming in Classroom Activities

Thesis for the degree *Philosophiae Doctor* (Ph.D.) at the
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Isebakke, Halden June 10, 2020 Sanna Forsström

Abstract

This thesis addresses programming integration in mathematics education by answering the question: what are the links between mathematics and programming in classroom activities? This thesis consists of three articles: one literature review and two empirical articles. The three articles are framed by an extended abstract.

The literature review discussing the educational potential of programming in mathematics education, brings out the need for discussing students' learning processes instead of learning results. There is a need for discussion about the influence of the teacher role and collaboration between students during the students' learning processes in programming activities. The two empirical articles address the students' collective learning processes in mathematics when Lego Mindstorms robots are introduced in a classroom of students aged 12 to 15, with a mathematics teacher who is a novice in programming.

The data from this study consists of ethnographic videotaped material of students' classroom activities with robots gathered in one lower-secondary school in Norway; field notes from the classroom observations; and, transcribed video material from the group interview with three students (the key informants). This data was analyzed with activity system analysis in Engeström's (1987) Cultural Historical Activity Theory (CHAT). The data analysis concentrated on students' activities with robots on a micro-level. The focus was on the relationship between different components during the activity development, such as the role of the teacher, collaboration between students, the object of the activity and the tools.

According to the findings of this study, mathematics can be linked with programming activities through the active and negotiating role of the teacher. These findings contribute to the debate in earlier studies regarding the transformational potential of digital technology in mathematics education. Earlier studies argued that it is unclear, how the potential links between digital technology and mathematics can be exploited. The findings contribute to this discussion by suggesting that the links between mathematics and programming activities have a transformational, yet not self-evident, potential in mathematics education. The study demonstrated that, during programming activities, mathematics could become the alive and

transformative object of the activity. The fruitful activity development took place when the teacher and the students collaborated.

Sammendrag

Denne avhandlingen handler om programmering i matematikkundervisning og tar for seg følgende problemstilling: hva er koblingene mellom matematikk og programmering i klasseromsaktiviteter? Denne avhandlingen består av tre artikler: én er en litteraturgjennom, mens de to resterende er empiriske artikler. I tillegg til artiklene består avhandlingen av et utvidet abstrakt.

Artikkel 1 gir en litteraturgjennomgang rundt det pedagogiske potensialet ved programmering i matematikkundervisning, samt viser frem behovet for å diskutere studenters læringsprosesser i stedet for læringsresultater. Det er behov for mer forskning innen påvirkningen av lærerens rolle, og samarbeid mellom elevene i deres læringsprosesser innen programmeringsaktiviteter.

De to empiriske artiklene tar for seg studentenes kollektive læringsprosesser i matematikk da Lego Mindstorm-roboter ble introdusert i et klasserommet for elever i alderen 12-15 år og hvor matematikklæreren er en nybegynner i programmering.

Dataene fra denne studien består av etnografisk datamateriale på video om elevenes aktiviteter i klasserommet med roboter på en ungdomsskole i Norge, feltnotater fra klasseromsobservasjonene, og transkribert videomateriale fra gruppeintervjuet med tre elever (hovedinformantene). Dataen ble analysert med aktivitetssystemanalyse i Engeströms (1987) Cultural Historical Activity Theory (CHAT). Dataanalysen konsentrerer seg på studenters aktiviteter med roboter på mikronivå. Fokuset var på forholdet mellom ulike komponenter under aktivitetsutviklingen, for eksempel lærerens rolle, samarbeid mellom elevene, verktøyene og objektet i aktiviteten.

Ifølge funnene kan matematikk knyttes til programmeringsaktiviteter gjennom lærerens aktive og forhandlende rolle. Disse funnene bidrar til debatt fra tidligere studier om teknologiens transformasjonspotensial i matematikkundervisningen. Tidligere studier hevdet at det er uklart hvordan potensielle koblinger mellom teknologi og matematikk kan utnyttes. Funnene i denne studien bidrar til denne diskusjonen ved å antyde at koblingene mellom matematikk og programmeringsaktiviteter har et transformasjonspotensialet i matematikkundervisning, men at det er ikke selvinnlysende. Det er vist i studien at under programmerings aktiviteter har matematikk en mulighet til å bli det "levende"

og transformative objektet i aktiviteten. En fruktbar aktivitetsutvikling fant sted når læreren og elevene samarbeidet.

List of publications

Forsström, S. E., & Kaufmann, O. T. (2018): “A Literature Review Exploring the use of Programming in Mathematics Education.” *International Journal of Learning, Teaching and Educational Research, 17*(12), 18–32. https://doi.org/10.26803/ijlter.17.12.2

Forsström, S. E. & Afdal, G. (2019): "Learning mathematics through activities with robots.” *Digital Experiences in Mathematics Education*. https://doi.org/10.1007/s40751-019-00057-0

Forsström, S. E. (2019): “Role of teachers in students’ mathematics learning processes upon the integration of robots.” *Learning, Culture and Social Interaction, 21*, 378-389. https://doi.org/10.1016/j.lcsi.2019.04.005

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Contents

Tables and figures:

PART I: Extended abstract

1. Introduction

1.1 Aim and the overall research question

“It was not allowed to use a mobile phone (in order to steer the robot),” one of the students answered when asked how the group of students got the idea to use touch sensors in order to control the robot. The students’ idea was innovative. Instead of programming each turn individually in different paths, the students wanted to make one universal program. The idea was that when the touch sensor was connected to the left-hand side and pressed down the robot would turn left; otherwise, the robot would drive straight forward. The right-hand side would work similarly. Figure 1 demonstrates the students installing these touch sensors and is an example from the data of this study where Lego Mindstorms robots, which can be programmed in the EV3-programming environment, were introduced to the students aged 12 to 15. The students were tasked to program the robot to drive a particular path; but, instead of programming the robot, the students were tempted to use an application on their smartphone to steer it. Based on the students’ earlier knowledge derived from outside of the classroom, they knew that the robot could be controlled with a smartphone application. However, the students were not allowed to use the application given that the idea was to program the robot. Thus, the students got the idea to make their own “application.”

Figure 1. The students installing and testing the touch sensors

According to Hoyles (2018), the integration of digital technology in mathematics classrooms has the potential to bridge classroom mathematics and the students' world outside of the classroom. During the session described above, the students indeed connected their activities with the robot to their "smartphone application world" outside of the classroom. Still, the links between classroom mathematics and the activities with the robot were not particularly visible during the session described above. However, the systematical use of mathematical tools in robot-based activities were visible in another session where students programmed the robot to drive a circle with a one-meter radius. In order to achieve this, they used proportions and circle geometry (see Figures 2 and 3). The students succeeded in their task (see Figure 4) and were very excited about their success (see Figure 5).

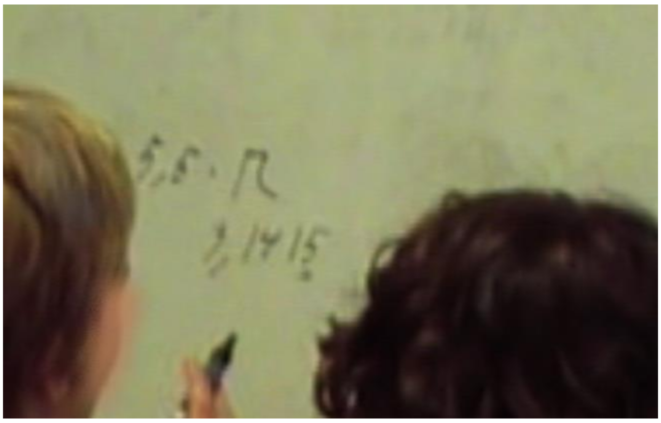

Figure 2. The student using mathematical tools in order to solve their problem

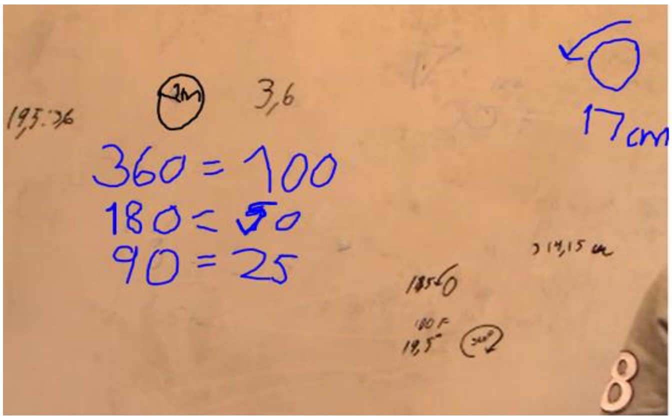

Figure 3. Reconstruction of what the students wrote on the whiteboard

Figure 4. The students succeeding with their task to program the robot to drive a circle with a radius of one meter

Figure 5. The students were excited when they successfully solved the problem

So, the links between robot-based activities and classroom mathematics were not self-evident. The links were constituted by many factors, such as the role of the teacher, collaboration between students and the tools which were used.

Computer programming, or coding, is defined in Balanskat and Engelhardt (2015) as a process of producing instructions in a programming language for a computer to do tasks, problem-solving and create interaction. Several countries, such as Finland and Sweden, have integrated programming in the mathematics curriculum (Bocconi, Chioccariello, & Earp, 2018). Norway is in the planning phase of doing so (Utdanningsdirektoratet, 2019). The majority of countries integrate programming in their curriculum in order to foster what is called students' 21st-century skills (Balanskat & Engelhardt, 2015). Desired such future skills, also referred to as 2030-century skills in the literature, relate not only to technological knowledge but also to social skills and creativity (Balanskat & Engelhardt, 2015; OECD, 2018).

In addition to this, the national curriculums, for instance in Finland, Sweden and Norway, call for connections between curriculum mathematics and programming activities (Opetushallitus, 2014; Skolverket, 2018; Utdanningsdirektoratet, 2019). However, the integration represents practical, everyday challenges firstly because a very new curricular activity is assumed to take time and energy from other activities in the mathematics curriculum. Secondly, because the role of the teacher may be challenged if the mathematics teacher does not have a relevant programming background (Bocconi et al., 2018), thirdly, it is unclear how programming can be linked to different subject areas (Balanskat & Engelhardt, 2015; Bocconi et al., 2018) in mathematics. Moreover, fourthly, it is also unclear how programming influences students' learning (Balanskat & Engelhardt, 2015; Bocconi et al., 2018) in mathematics.

This study is an empirical contribution to the knowledge of the potential links between curriculum mathematics and programming activities in lower secondary classrooms. It discusses practical, everyday situations in the classroom and considers the possible challenges in programming integration, such as its connections with the mathematics curriculum and the role of the teacher. This study aims to answer the overall research question:

What are the links between mathematics and programming in classroom activities?

The relevance of the findings of this study will be discussed by addressing issues in a traditional mathematics classroom. One critical issue in mathematics education is the difficulty in motivating students to learn formal curriculum mathematics. Hoyles (2018) pointed out that students often connect curriculum mathematics with abstract procedures and rules without making connections outside of the classroom. However, the students' motivation to learn mathematics derives from the role of mathematics in the students' lives outside of the classroom. Students sometimes have difficulties in understanding the importance of learning mathematics, if the mathematics they have to learn does not have any concrete role in their lives (Gellert & Jablonka, 2009; Gura, 2007; Hoyles, 2016).

Computer programming falls under the umbrella of digital technology. According to Hoyles (2018), integration of digital technology has the potential to transform learning and teaching activities in mathematics classrooms by providing outside-of-classroom connections for mathematics. Hoyles (2018) argued that digital technology as a tool has the potential to enhance the students' conceptual engagement in mathematics. However, Drijvers (2018) responded to Hoyles (2018) by claiming that this is not that self-evident, because “not enough is yet known about how to exploit this link between mathematics and tool use, and the way in which this transforms practices for the sake of mathematical learning” (Drijvers, 2018, p. 230). This study contributes to this discussion by addressing the potential links between curriculum mathematics and programming activities in the everyday classroom context.

1.2 My background as a mathematics teacher

My interest in this topic stems from my background as a mathematics teacher. I have over 10 years of experience in teaching mathematics in Finland and one in Sweden. This experience was for different schools and school levels. Throughout my work experience, I have met different kinds of students, become familiar with different types of learners and seen many different learning environments. Despite school levels or students, the same challenge remained present: how can I motivate students to learn mathematics. Conversely, the students themselves also had a recurring question: for what purpose do we need this? As a mathematics teacher, to answer their question, I referred to activities outside of the classroom in order to motivate students to learn mathematics. However, I found it challenging to find connections, which are purposeful for students.

Before moving to Norway and embarking on my Ph.D. studies, I was working as a mathematics teacher in one lower-secondary school in Finland. Coincidentally, Finland was in the planning phase of integrating programming as a part of the mathematics curriculum during the last couple of my teaching years. The planning included teacher courses, and many of the schools were ready to test programming. I attended an introduction day for the teachers; and, after a short introduction of Lego Mindstorms robot, I was excited to test them with my students aged 12 to 15. The students and I tested the robots together in the mathematics classroom for two years. The approach was very student-centered: students were able to find their projects and tasks. In turn, as the teacher, I supported them, guided them, and helped them with programming when needed. These robot-based activities took place alongside other activities in the classroom and were mostly extracurricular. Although I saw robot-based activities as an excellent supplement for traditional classroom activities, I did not see the direct connection with curriculum mathematics and robot-based activities.

To illustrate this thought process, I use the example of a student I taught whom I will henceforth refer to as Pekka. Pekka was not motivated by curriculum mathematics. I introduced Lego Mindstorms robots in his class with the idea that we would learn to handle them together. Students had time to become familiar with robots alongside traditional school mathematics. They were able to create their ideas about what to do with the robots working in teams of two or three students. So, Pekka and his team decided to build a car and start programming it. Pekka took charge when they were programming, and after a couple of attempts, they started to program their car to do pocket parking. They used trial and error to achieve their goal. After each trial, the students negotiated with each other and shared ideas about the next steps and how to move forward. There was also a time to negotiate the point at which the project was deemed successful, or “ready,” given that this could be a never-ending problem; you can always be a bit better at parking.

The point I do want to make with the case of Pekka is that even though I had never seen Pekka as excited and spontaneous with many ideas and thoughts in the mathematics classroom as he was with this task, I had difficulties, as a mathematics teacher, to see the links between curriculum mathematics (my primary concern) and robot-based activities. Even if I, as a mathematics teacher, saw how students found robot-based activities motivating, I did not see connections between these two different components in my classroom. Thus, according to my preconceptions based on my teaching experience, I was curious but at the same time skeptical about the usefulness of robots and programming in mathematics education.

1.3 Literature review (Article 1): research on the educational potential of programming in mathematics education

In order to discern what earlier studies have revealed about the potential links between the mathematics and programming activities, a literature review of research was conducted, together with Odd Tore Kaufmann, relevant to the following question:

What is the educational potential of programming in mathematics education?

From the results of our systematic search, we identified and analyzed 15 articles with different study types, themes, and designs. Based on these, we identified three dominant themes: (1) the motivation to learn mathematics, (2) student performance in mathematics, and (3) the collaboration between students and the transforming role of the teacher. According to the results of these studies, programming integration improves students' motivation to learn mathematics and students' performance in mathematics for some of the groups of students. We concluded that earlier studies concentrated mostly on individual learning results and motivation. Although the articles reported that collaboration between students was widely used in programming activities and that the role of the teacher was different than before the integration of programming in the classroom, the influence of these components for students' learning was not discussed. We argued that there is a need for studies analyzing students' collective learning processes, instead of relying on research that sees individual learning results as sole indicators, in order to get more detailed information about the potential links between curriculum mathematics and programming integration. Collaboration between students has been widely used in programming activities, and the role of the teacher has also been found to differ from traditional; therefore, discussion is needed about the effect of these components in students' learning processes (Article 1).

Having reviewed the literature, we searched for a more detailed understanding of the links between curriculum mathematics and programming activities by concentrating on analyzing students' collective activities with robots on a micro-level in order to understand their learning. We viewed learning as something that can be understood through analyzing collective processes instead of merely attending to results on paper-and-pencil tests. We concentrated on interactions between students, teacher, programming tools and robots.

1.4 Research questions, articles, and the research strategy

In order to find out more about the potential links between curriculum mathematics and programming activities in the classroom, I conducted a study, with the features of an ethnographic and intervention study, in one lower-secondary classroom in Norway, given that Norway is planning to integrate programming into their mathematics curriculum. This study corresponds to the everyday situation where the mathematics teacher, who does not have a relevant programming background, integrates programming in their classroom. One way to make programming integration somewhat smoother when the teacher does not have extensive training in programming is to use visual programming environments to get started (Bocconi et al., 2018). Thus, this study concentrates on the integration of Lego Mindstorms robots in a classroom, where the teacher and the students did not have any extensive training. As previously mentioned, Lego Mindstorms robots can be programmed in the EV3-programming environment; functionally, this means that the steering of robot motors and utilization of different sensors take place through different visual blocks (Bocconi et al., 2018).

This study has ethnographical features with me as the researcher in the same social space (classroom) as the students and teacher. More specifically, the research strategy in this study is called a focused ethnography, with focused observations, key informants, and more time-intensive fieldwork than in a traditional ethnography (Skårås, 2018). Also, my role as a researcher differed from what is the case in traditional ethnography. I introduced Lego Mindstorms robots shortly for the mathematics teacher; and thus, this study has also features of an intervention study. Following this, the teacher introduced robots in their elective study classroom for students aged 12 to 15.

Moreover, elective study program was chosen because the free environment of a class activity provided an opportunity for innovative learning processes without any pressure from the mathematics curriculum. I followed the students working with robots by videotaping the activities and writing field notes during one semester, once a week during 75-minute sessions. That was the time needed to gather insights into the use of mathematics in the classroom. My observations concentrated on the key informants, one group of three students, Jacob, Lucas, and Oscar, aged 12 to 13, and their learning activities with robots. The specific focus on one group of students made it possible to gain an understanding of the learning processes on a micro-level. The micro-level observations made it possible to analyze students' activities in detail by focusing on students' communication and interactions with different gestures. This gave valuable information about the links between mathematics and robot-based activities.

In order to get a more detailed understanding of the potential links between mathematics and robot-based activities, the overall research question is discussed in more detail in two additional articles with two separate research questions (see Table 1). The discussion about mathematics in use and the role of the teacher are thus divided into two different articles. Even if articles 2 and 3 discuss the same students and teacher, these articles focus partially on different sessions. Moreover, the articles focus on different components in robot integration. The focus in Article 2, "Learning Mathematics Through Activities with Robots," is on mathematical tools in use and components influencing it, coupled with activity development in students' learning processes with mathematical tools in use. The focus in Article 3, with the title "Role of Teachers in Students' Mathematics Learning Processes upon the Integration of Robots," is the role of the teacher in the students' learning processes. So, articles 2 and 3 contribute to different discussions.

The links between mathematics and robot-based activities are discussed in Article 2 with the help of the concept of tool in Cultural Historical Activity Theory (CHAT). I will present CHAT in more detail in the following subsection entitled Theory and

Methods, and in third chapter, entitled Theoretical Framework; but, shortly, the central unit of analysis in CHAT is a tool mediated collective activity system. The concept of object has a central role in activity system analysis as a motive and direction for the collective activity (Engeström, 1987; Roth & Radford, 2011). The use of tools is constituted[1] by the object, the drive, or direction of the activity. Thus, the focus in Article 2 is on the relationship between mathematical tools and object of the activity. The article answers the question: *what is the relationship between mathematical tools and object in robot-based collective student learning activities in secondary education?*

Article 3 contributes to the discussion about the role of teacher in fruitful integration of digital technology, and the potential of educational technology to change the mathematics classroom into a more student-centered one. According to earlier studies, integration of digital technology has the potential to make mathematics classrooms more student-centered and to increase the motivation of students. However, the change depends on the role of the teacher (Bray & Tangney, 2017; Olive et al., 2010). Moreover, Article 3 discusses the situation where the teacher does not have extensive training in programming. It answers the question: *how does the role of the teacher in robot-based activities influence students' learning processes in mathematics?* The focus is also on the object of activity, which is discussed more in detail in the following subsection. As Article 2 discusses the relationship between tools and object, Article 3 discusses the influence of the role of the teacher on the objects of students' activities with the robots.

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[1] I use the concept of constitute to describe relationships that are not causal but mediating.

Table 1. The articles and research questions of this study

	Article 1: Literature Review	**Article 2: The Use of Mathematical Tools**	**Article 3: The Role of the Teacher**
Title	A Literature Review Exploring the Use of Programming in Mathematics Education.	Learning Mathematics Through Activities with Robots.	Role of Teachers in Students' Mathematics Learning Processes Based on Robotics Integration.
Research question	What is the educational potential of programming in mathematics education?	What is the relationship between mathematical tools and object in robot-based collective student learning activities in secondary education?	How does the role of the teacher in robot-based activities influence students' learning processes in mathematics?
Topics	Motivation to learn mathematics; students' performance in mathematics; the collaboration between students; the role of the teacher; curriculum connections with mathematics.	The use of mathematical tools; the object of the activity; connections with mathematics curriculum.	The role of the teacher; the collaboration between students; object of the activity; mathematical and technological tools.
Contributes to the discussion about	The article focuses on the potential benefits of programming in mathematics education and unsolved questions regarding them.	The article focuses on the connections between robot-based activities and mathematics curriculum, along with collective learning processes in mathematics through activities with robots.	The article focuses on the role of the teacher in a fruitful robot integration, and the role of the teacher in not a teacher-led classroom when the teacher does not have any programming background.

1.5 Theory and methods

In order to understand collective learning processes, this study is based on the socio-cultural paradigm, which means that knowledge creation is viewed as a social process with different tools in use (Vygotsky, 1978). Based on the socio-cultural perspective, the focus of this study is on how students and the teacher use different kinds of tools. According to the socio-cultural perspective, knowledge, and skills stem from patterns and insights created over time in societies (Säljö, 2014). Thus, knowledge creation processes emerge through social interactions, which enable involvement in historically accumulated cultural patterns and tools, such as different kinds of psychological, linguistic or physical tools (Säljö, 2014). This means that learning in this study is seen as a social process involving the use of different kinds of tools. Knowledge creation processes relate to social interactions and argumentation.

More specifically, the knowledge creation processes in this study are discussed with Engeström's (1987) Cultural Historical Activity Theory (CHAT), where the knowledge creation processes consist of social, multi-voiced interactions. CHAT is useful for this study because the focus is on the collective classroom activities instead of on individual actions. The analysis of collective activities gives the possibility to get information about relational processes in the classroom, which is difficult with a more individual starting point. CHAT makes it possible to analyze the role of mathematics and digital technology as part of classroom activities instead of just an external object or tool. Furthermore, learning processes in the classroom are discussed without analytical distinction between teacher and students; CHAT sees teaching and learning as dialectically intertwined processes (Engeström & Sannino, 2012). The teacher is a part of students' learning processes, which gives information about teacher-student relationships in the classroom. Thus, CHAT suits well with analyzing collective learning processes in the context of robot integration. At the beginning of the programming integration, frequently both students and the teacher are novices in programming; and so, learning activities are often collective. In that kind of situation, the teacher may also take the role of a learner, if they have insufficient knowledge in programming.

In order to analyze and understand the learning processes described above in detail, I used the activity system model (see Figure 5) in Engeström's (1987) CHAT, where the seven components (see Table 2) describing the collective activity are in a relationship with each other through mediation (Engeström, 2005). For instance, in the uppermost sub-triangle, *tool* mediates the *subject*'s activity towards *object*. With the help of tools, subjects are in interaction with the object of the activity (Engeström, 2008). The social components, rules, community and division of labor influence collective activities. These components make it possible to address collective achievements (Engeström, 2008).

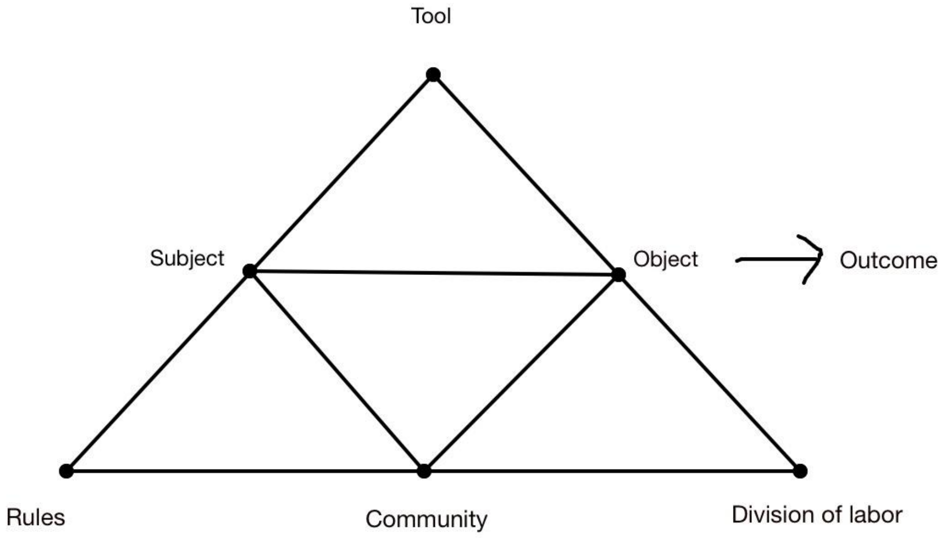

Figure 6. The Activity System Model (Engeström, 1987, p.78)

Table 2. Components in the Activity System Analysis (retrieved from Article 2)

Component	**Definition/meaning**	**Examples from this study**
Subject	The individual or group of people who are engaging in the activity (Yamagata-Lynch, 2010)	The students and the teacher
Object	The driving force of the activity (motive and goal) (Engeström, 1987)	Fulfill a task with the robot
Tool	The instrument that mediates the activity (Engeström, 1987)	The robot, computer, and mathematical tools
Rules	The regulations that are relevant to the activity (Yamagata-Lynch, 2010)	Task assignment, the rules of the mathematics classroom
Community	The social group to which the subject belongs to during the activity (Yamagata-Lynch, 2010)	The whole class of students and the teacher (or teachers)
Division of labor	How the tasks are shared during the activity (Yamagata-Lynch, 2010)	Collaboration between students, the mediation of the teacher
Outcome	The result of the activity (Yamagata-Lynch, 2010)	The robot drives a track as it is programmed

Because this study concentrates on the role of the teacher in students' learning processes and links between mathematical tools and the activities with robots, the focus is specifically on the components of *tools*, *object,* and *division of labor*. The use of mathematics and other tools is constituted by the object of the activity, which is the foundation for the whole activity (Engeström, 2005). The role of the teacher can be discussed through division of labor in the activity system analysis.

1.6 The study design

According to Maxwell (2005), the design of a qualitative study consists of interactions between its goal, research questions, theoretical framework, methodology, and validity. The design of this qualitative study with connections between research questions, theoretical framework and a research method are summarized in Figure 7. This study can be classified under the sociocultural paradigm, where the theoretical framework of this study, CHAT, belongs also. In the sociocultural paradigm, knowledge creation is seen as a social process with cultural mediation, i.e. with cultural tools in use (Säljö, 2014).

Furthermore, the overall research question is discussed with the help of the components from the activity system analysis in CHAT. The most central concepts are learning processes, tools, object, the role of the teacher, and collaboration. The need for discussion about these concepts stems from the literature review article, which emphasized the need to discuss collective learning processes in mathematics and the role of the teacher in them, instead of focusing solely on individual learning outcomes. The focus is on the relationships between the teacher, the tools, the object of the activity and division of labor in students' collective learning processes. Article 1 discusses students' learning and motivation to learn mathematics. Articles 2 and 3 both discuss students' collective learning processes. In Article 2, the focus is on mathematical tools in use and students' collective learning in mathematics. Article 3 focuses on the role of the teacher in students' collective learning processes. The most central theoretical concepts in articles 2 and 3 are the concepts of tools and object. Figure 7 shows how these different themes and components, mathematics learning, motivation to learn mathematics, collaboration between students and the teacher, the roles of the teacher are presented in different articles in this study. Figure 7 also shows how different articles in this study are connected through these themes and components.

Articles 2 and 3 fit in the sociocultural paradigm. The focused ethnographical data gathered in order to answer the research questions in articles 2 and 3 have been analyzed with the help of CHAT.

As the design of this study is a complex interactive model, where each part depends on each other, I will discuss the connections with other components in my study design later in this thesis. In the section of a theoretical framework, I will reflect in more detail on the usefulness of CHAT in this study. In the methodology section, I will present a more detailed research strategy, a focused ethnography. I will present my data and methods more detailed and reflect, how CHAT and focused ethnography influenced my data collection and analysis methods.

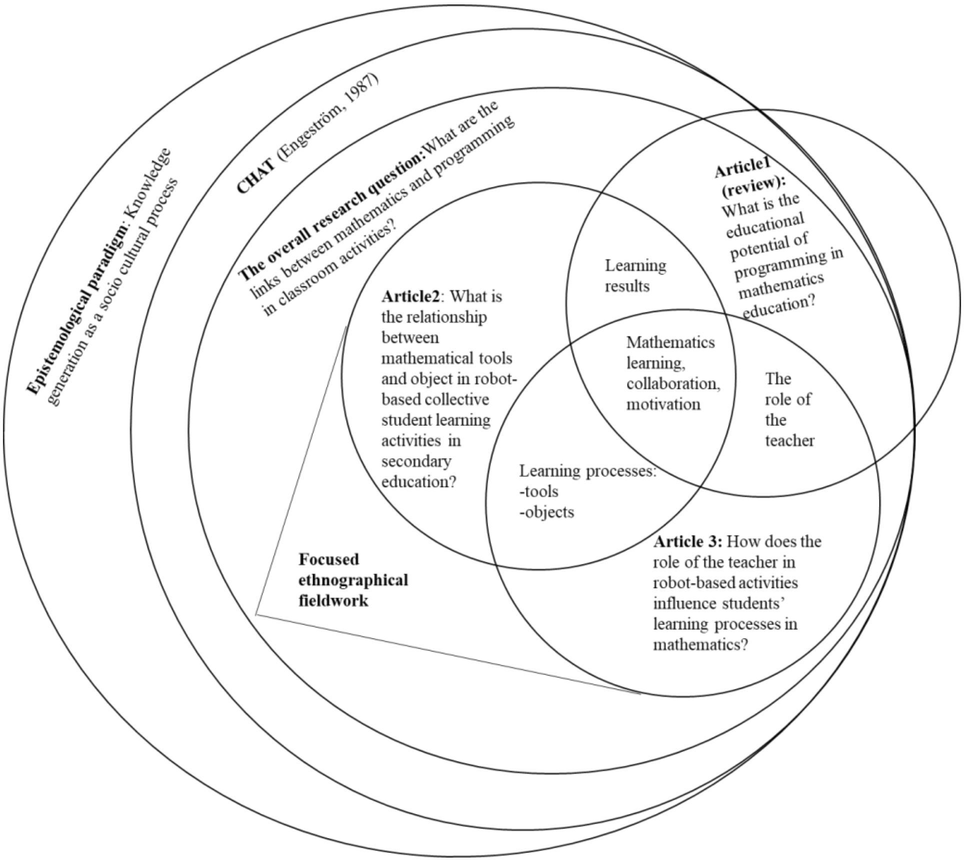

Figure 7. Research design

1.7 Outline of the thesis

This thesis consists of two parts. The first part clarifies my overall research question, the aim and the context of the study. Here, the theoretical framework and methodological approach, as well as the contributions of this study, are discussed in detail. The second part consists of three articles: one literature review article and two empirical articles.

In the first part of this thesis, the second chapter, I discuss in which different contexts I place this study. In the third chapter, I introduce the theoretical framework used in this study. I discuss how different components and characterized features of CHAT suit this study. In the fourth chapter, the methodological parts of this study will be presented. The fieldwork based on focused ethnography and my role as a researcher will be presented in detail, the methods of analysis, and the reliability and validity of this study will be discussed. The fifth chapter is a summary of the two empirical articles. In the last chapter, I provide a discussion about the overall contribution of this thesis towards the field. The overall research question will be discussed and answered with the help of the contributions of all three articles.

The second part of this thesis consists of the three articles as separate appendices. These three different articles contribute to the discussion on the links between mathematics and programming activities independently from three different viewpoints.

2. Context

In this chapter, I will introduce the context in which this study is situated. As discussed in the Introduction, this study focuses on collective classroom activities, and the knowledge-creation processes herein are discussed through the lens of Engeström's (1987) Cultural Historical Activity Theory (CHAT). Thus, I will describe the context of this study by first discussing, how context is understood in CHAT.

A context is often understood to be a predefined and stable frame of a study that is "out there," but CHAT understands contextualization as being a dynamic process. As Nardi (1996, p. 38) stated:

> Context is constituted through the enactment of an activity involving people and artifacts. Context is not an outer container or shell inside of which people behave in certain ways. People consciously and deliberately generate contexts (activities) in part through their own objects; hence context is not just "out there."

As a theoretical framework of this study, CHAT is discussed in greater detail in the following chapter, so without any deeper explanation here, I will only mention that CHAT is characterized by a procedural and a relational perspective (Nardi, 1996), which means that "context" in CHAT is understood more as a process than as a given product. In CHAT, context forms in the different relationships within an activity. In this study, the context is formed in classroom activities, which extend far with time and space and are in relationship with other activities (Nardi, 1996; Van Oers, 1998); these activities can still be framed by certain relationships.

Again, Nardi (1996, p. 38) wrote:

> A context cannot be reduced to an enumeration of people and artifacts; rather the specific transformative relationship between people and artifacts, embodied in the activity theory notion of functional organ, is at the heart of any definition of context, or activity.

As the focus in this study is on collective classroom activities, the context of this study is formed in different transformational relationships in these activities and in transformative relationships between people and tools (i.e., artifacts) in these activities. Even if the context forms within activities, those activities, the participants, and the tools carry their own histories based on other contexts. Those classroom activities are in relationship with other activities, both inside and outside of the classroom, which again, can be connected with many different contexts, such as curriculum, tests and grades, society, political decisions, and media. In order to understand collective classroom activities and the transformations and relationships within the activities and between different activities, the activities need to be connected with different kinds of contexts inside and outside of the classroom.

Thus, the activities inside the classroom need to be discussed in a wider context, and this study needs to also be connected with contexts other than just classroom activity. On the other hand, as the study needs to be located in time and space, all possible connections or relationships cannot be analyzed and discussed (H. Afdal & Afdal, 2010). H. Afdal and Afdal (2010) pointed out that the context in educational research is constituted by the research design and has four actors: the studied object, the researcher, theory, and the research participants. These actors frame the study with time, space, and relations and thus make the study manageable.

When contextualizing this study, I, as a researcher, created the context of this study in a dialogic process with the four actors: the classroom activities as the studied object, the research questions formulated by me as a researcher, CHAT as a theory, and the teacher, together with the students, as research participants. In this inner-dialogic process with me as a researcher, my preconceptions and interests were challenged by earlier studies and the data material of this study. Thus, based on the dialogic process with the research questions and observed classroom activities, I made choices regarding connections and relationships addressed in this study. I based my decisions on the research questions, the research field, earlier studies, and practical solutions regarding study design. These frameworks make the study manageable and conductible.

Based on the dialogic process with research questions and classroom activities, the contexts established further in this chapter and in this study are as follows: 1) The research field of and critical issues in mathematics education; 2) Digital technology in mathematics education; 3) Robots in mathematics education; and 4) Mathematics and digital technology in Norwegian schools. I will justify the choice of these contexts henceforth.

First, in order to justify Contexts 1 and 2: Based on the research questions, I was out to find a more detailed understanding of the potential links between mathematics and programming activities. At the activity level, I was interested in learning more about the role of mathematics and the role of programming in classroom activities. The discussion about the role of the mathematics in classroom activities is placed in the research field of mathematics education. As the focus is also on the role of programming in classroom activities and potential links between mathematics and programming activities, the discussions in this study focus on the research field of digital technology in mathematics education (i.e., Context 2). Furthermore, as I argued in the Introduction, it is unclear how the potential links between mathematics and programming activities transform activities in the classroom and influence students' learning processes in mathematics; hence, this study focuses on transformative learning processes with digital technology in the classroom. The discussions concentrate on the relationships between different components, such as the role of the teacher, collaboration between students, the role of programming/robots and the role of mathematics in the collective classroom activities and the activity development during these activities. In order to discuss the relevance of the findings of the discussions about how the role of mathematics and that of the teacher in students' programming activities are connected to the research field of digital technology in mathematics education by also discussing critical issues in mathematics education (i.e., Context 1). As discussed in the Introduction, the earlier studies have connected the integration of digital technology with critical issues in mathematics education and discussions on how the integration of digital technology can contribute through transformative classroom activities to reduce these issues.

Second, the Lego Mindstorm robots present one way to connect programming in classroom activities (i.e., Context 3). Robots also bring a more concrete dimension to programming and Lego Mindstorm robots are the most studied educational robots (Benitti & Spolaôr, 2017).

Third, programming is a current topic in mathematics education in many countries, especially in Norway at the moment. After the curriculum reform in Norway, programming is going to be a part of students' and teachers' activities in mathematics classrooms. Thus, the data collection of this study took place in a classroom in a Norwegian school context (i.e., Context 4) with students and the teacher as research participants and their activities with robots as a unit of analysis.

The programming topic could be approached from several different angles and research environments, such as by discussing curriculums, students outside of classroom activities or students' improvements in mathematics. However, such approaches would require a different kind of study design with different kinds of practical solutions. I found it difficult, for instance, to follow students outside of the classroom activities, because I did not know the informants of this study beforehand. The framing of the study with time and space would also prove to be challenging. The programming activities in a Norwegian classroom environment provide the possibility to discuss what kind of learning processes are activated when programming as a new element comes into play.

Thus, different contexts discussed here are issues in mathematics education, digital technology in mathematics education, and robots in mathematics education. The Norwegian school context is also presented.

2.1 Issues in mathematics education

Mathematics is an integral part of our lives and our society; mathematics is everywhere. In research, for instance, it is not only needed when modeling in the sciences, the social sciences, and in economics, but also many areas of the humanities. In our everyday lives, on the other hand, we experience it through its role in tools, such as computers and smartphones. However, students still have difficulties understanding the importance of learning mathematics, even if mathematics education is under development, and even though mathematics is present in digital technology and society (Gellert & Jablonka, 2009; Gura, 2007; Hoyles, 2016). Even though mathematics is everywhere, mathematics is mostly invisible in our society (Hoyles, 2015). Several researchers argue that, even though mathematics is everywhere, mathematics education is not connected with everyday activities outside of school or other school subjects in many classrooms (Boaler, 2009; Bray & Tangney, 2017; Hoyles, 2016). In this context, mathematics classrooms are often quite teacher-centered, and teachers give students ready-made tasks to solve (Engeström, 2008; Olive et al., 2010; Valoyes-Chávez, 2019).

An additional issue in a traditional teacher-led mathematics classroom is the complexities of teaching and learning practices (Hoyles, 2015). According to reform suggestions in mathematics education, an effective approach in the classroom is student-centric, and teaching and learning activities are interdependent processes (Franke, Carpenter, Levi, & Fennema, 2001; Valoyes-Chávez, 2019). This kind of approach is called reform mathematics, which utilizes a problem-oriented and inquiry-based approach (Goos, 2004), wherein the teacher meets students' needs based on their own interests and previous knowledge (Valoyes-Chávez, 2019). In reform-oriented, inquiry-based classrooms, the aim is to get students to participate and engage in the communication, reasoning and problem-solving activities inside the classroom (Goos, 2004). However, it is not self-evident, "what kinds of practices do we wish students to participate; and … what specific actions … a teacher [should] take to improve students' participation" (Goos, 2004, pp. 281–282). Furthermore, an additional issue is how the teacher is able to connect activities in the classroom with students' lives and cultures outside of the classroom (Goos, 2004; Parker, Bartell, & Novak, 2017).

Despite curriculum reforms and pedagogical discussions about practical problem-solving activities and integration of digital technology into school mathematics (Albert & Kim, 2013; Contreras, 2014), a traditional teacher-led approach connected with abstract mathematics still remains to be established in an everyday context in many mathematics classrooms (Albert & Kim, 2013; Bray & Tangney, 2017; Opheim & Simensen, 2017). A broad range of research argues that students connect the subject of mathematics with abstract procedures, rules, and memorization, all of which have a central role in the mathematics classrooms (Albert & Kim, 2013; Bray & Tangney, 2017; Hoyles, 2016, 2018; Opheim & Simensen, 2017; Pietsch, 2009). In this context, students often have a passive relationship with their learning, they follow fixed rules and get fixed, unquestioned answers with fixed numbers (Boaler, 2009; Bray & Tangney, 2017; Ernest, 1996; Hoyles, 2016).

This kind of traditional classroom environment, which is usual in mathematics education, can be illustrated with Engeström's (1987) activity system model. In a traditional classroom, the teacher and students have different activity system models (see Figure 8). In a student's activity system model, the object of the activity is traditionally the task given by a teacher, often from the book. The desired common outcome in both activity systems is test results and grades. This is the only shared component in the teacher's and the student's activity system models. (Engeström, 2008). Engeström (1987) calls objects, which are reproduced in order to gain test results or grades, dead objects (Engeström, 2008).

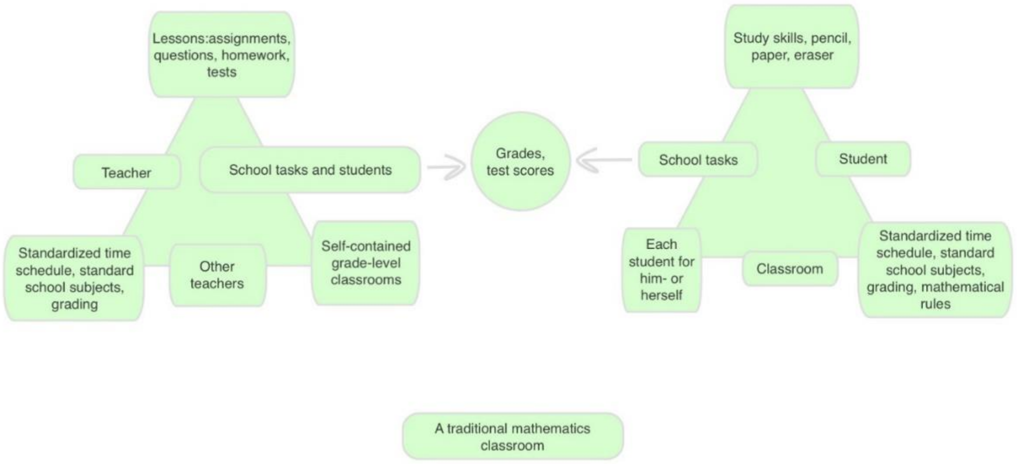

Figure 8. The model of traditional mathematics classroom, reconstructed from Engeström (2008, p. 89).

To interpret the description above concerning the situation where mathematics is connected with procedures and rules with Engeström's (2008) traditional classroom model (see Figure 8), mathematics has a role as rules or pre-defined tools in a student's activity system model. Students are assumed to use specific mathematical tools with certain rules in order to solve tasks given by the teacher.

Engeström (2008) argues that when discussing the potential to change classroom culture, the focus should be on the object of education. According to Engeström (2008), the potential changes in the classroom activities occur through object constructions, which are influenced by all components in the activity system model. Thus, it is interesting to uncover whether or not the integration of robotics in mathematics classrooms has the potential to make changes to the objects of activities; and if so, how? From a broader perspective, according to earlier studies, integration of digital technology, in general, has the potential to change a traditional classroom culture in mathematics education. This potential will be discussed in the following section.

2.2 Digital Technology in Mathematics Education

Digital technology, such as computers, tablets with different applications, and digital whiteboards, have been a long-standing staple of mathematics education. This idea to integrate programming in mathematics classrooms is not new. As Papert suggested in 1980, programming should be included in the school curriculum. He argued that mathematics teaching and learning often happens through primitive technological tools, such as paper and pencil, for instance, used in drawing different graphs or solving equations with a set of guiding principles. This kind of teaching or learning practices is not connected to the students' world outside of the classroom. Papert's (1980) idea was to reconstruct learning and teaching processes in mathematics through programming integration by providing new relationships and connections with formal mathematical knowledge. According to his suggestions, mathematical activities through computer programming could transform the roles in a traditional teacher-led classroom. As he stated it:

> Mathematics is a real activity that can be shared by novices and experts. The activity is so varied, so discovery-rich, that even on the first day of programming, the student may do something that is new and exciting to the teacher. (Papert, 1980, p. 179)

After Papert's suggestion, a lot has happened in mathematics classrooms with developing digital technology in the classroom. Still, as discussed in the previous chapter, the traditional classroom model is usual in many mathematics classrooms. The discussion about the usefulness of digital technology in mathematics education is still ongoing, and it has taken until now to integrate programming into the school curriculum of many countries.

According to earlier studies, the integration of digital technology has a potential to change the traditional teaching and learning processes in mathematics education by offering connections outside of the classroom and possibility for students to get ownership of their learning (Bray & Tangney, 2017; Gellert & Jablonka, 2009; Hoyles, 2016, 2018; Olive et al., 2010). However, that is not self-evident. The links between curriculum mathematics and technology-based activities in the classroom are not always that simple (Drijvers, 2018). Furthermore, a range of earlier studies argued that how we integrate digital technology in the mathematics classroom is critically determined by the teacher’s complex role in the classroom and their knowledge (Drijvers, 2012, 2015; Drijvers et al., 2014; Ruthven, 2014).

A number of earlier studies approach the teacher’s role and knowledge during integration of digital technology with the theoretical concept of instrumental orchestration (Drijvers, 2012; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Drijvers et al., 2014; Ruthven, 2014) developed by Trouche (2004) and/or the TPACK (Technological Pedagogical Content Knowledge) model (Drijvers et al., 2014; Ruthven, 2014) developed by Mishra and Koehler (2006). These approaches focus on teacher’s expertise in the classroom (Ruthven, 2014). As the studies using the concept of instrumental orchestration focus on the teachers’ classroom practices with digital technology, the TPACK model is used to discuss the teachers’ skills (Drijvers et al., 2014).

Trouch (2004) defined instrumental orchestration as the teacher’s organization and management of the use of technological artifacts (i.e., tools) in the classroom in order to steer students’ instrumental genesis, which is a process wherein artifacts are distinguished from instruments; artifacts are something that has been given, and an artifact can become an instrument when a subject applies the artifact in her activities. Trouche (2004, p. 285) defined the concept of instrument and construction thereof as follows: “An instrument can be considered as an extension of the body, a functional organ made up of an artifact component (an artifact, or the part of an artifact mobilized in the activity) and a psychological component. The construction of this organ, named instrumental genesis, is a complex process, needing time, and linked to the artifact characteristics (its potentialities and its constraints) and to the subject’s activity, his/her knowledge and former method of working.”

In the instrumental genesis, “tool and person co-evolve so that what starts as a crude ‘artefact’ becomes a functional ‘instrument’ and the person who starts as a naive operator becomes a proficient user” (Ruthven, 2014, p. 7).

According to the studies using the concept of instrumental orchestration, integrating digital technology into the mathematics classroom may transform teaching practices, but it is hardly a revolution (Drijvers et al., 2010). When discussing the different kinds of instrumental orchestrations used by a teacher in a technology-rich classroom, Drijvers et al. (2010) divided teachers' different orchestrations into three different categories: *a didactic configuration*, *an exploitation mode,* and *a didactical performance*. A didactic configuration handles the ways in which teaching is didactically organized in the classroom; for instance, which artifacts (tools) are planned to be used in education. An exploitation mode handles, for instance, decisions about different tasks or about how artifacts (tools) are intended to be used. A didactic performance handles decisions made by a teacher in a teaching situation in the classroom. Drijvers et al. (2010) argued that when new digital technologies are introduced in the classroom, the teachers' new practices and didactic decisions are related to the practices with which they are familiar: the teachers make their own choices which are related to their regular habits. The teacher-led approach with closed questions is still more or less present in teaching practices with digital technology (Drijvers et al., 2014).

Moreover, according to Drijvers et al. (2014), the teachers' skills are an essential factor in the successful integration of digital technology. Drijvers et al. (2014) argued that teachers use their pedagogical-content knowledge alongside their technological skills, which enables satisfactory integration of this technology in most of the cases. Still, the teachers who are less experienced and skeptical of the use of digital technology are an essential group to consider when discussing the potentials of technology integration. I think this is an interesting point to consider when discussing programming integration in mathematics education, when mathematics teachers do not necessarily have any extensive training in programming.

The role of the teacher may also become challenging when linking digital technology-based activities with curriculum mathematics. Earlier studies argued that the students' focus is often on the technological and practical side, while mathematics teachers' concerns lie mostly on mathematics (Drijvers et al., 2010; Lagrange & Ozdemir Erdogan, 2009). When students need much technological advice from the teacher, the focus on teaching practices is mostly on the digital technology itself, rather than on mathematics (Drijvers et al., 2010). Drijvers et al.

(2014) found that after the resolution of technological issues, there is a place for mathematics, which creates pressure on the teacher to develop sound technological skills.

In order to summarize the challenges regarding the transformative potentials of digital technology integration in mathematics education, I interpret Drijvers' (2018) response to Hoyles' (2018) optimistic ideas about the potentials of digital technology. Drijvers (2018) claimed that it is not clear enough how the potential links between mathematics and digital technology-based activities can be exploited to transform processes in the mathematics classroom. His first concern was the *why* question. He claimed that if the reason to transform learning and teaching practices in mathematics classrooms is to foster students' measurable outcomes, there is not enough research-based evidence to do so (Drijvers (2018).

Drijvers' (2018) second concern was the *into what* question. He claimed that it is unclear into what teaching and learning practices should be transformed. Hoyles (2018) argued that digital technology could provide new windows in mathematics learning when students are using mathematics in new situations. However, as discussed earlier in this section, the mathematics behind the digital technology is not the students' first concern when they are using the technology. The teacher is more concentrated on mathematics than students. Thus, the links between digital technology-based activities and mathematics are still unclear.

Drijvers' (2018) third concern regarding the transformative power of digital technology is *by whom*. Hoyles (2018) suggested that technological tools can transform classroom practices. However, Drijvers (2018) argued that tools alone could not make any changes; the potential of digital technology is determined by the manner in which it is used or planned-to-be-used by teachers or educational designers. As discussed earlier in this section, teachers' practices and skills influence in which manner digital technology will be integrated.

2.3 Robots in mathematics education

There are many different robots or toolkits which can be used educationally (Karim, Lemaignan, & Mondada, 2015), and Lego Mindstorms robots are the most studied (Benitti & Spolaôr, 2017).

This study concentrates on Lego Mindstorms robots because they provide a smooth way of integrating programming in the earlier stages of integration. The visual programming environment, where different figures represent different programming structures (such as loops and if-structure) make the teaching of programming easier for teachers, who do not have any extensive training in programming (Bocconi et al., 2018). In the EV3-programming environment, it is possible to program Lego Mindstorms robots by changing the values of different variables in different figures (Bocconi et al., 2018).

Educational robotics provides an opportunity outside of the classroom connections in mathematics education (Ardito, Mosley, & Scollins, 2014; Barak & Assal, 2018); but, as discussed, the potential of digital technology in mathematics education is not self-evident. Moreover, the educational benefits and curriculum connections with robotics remain unclear (Alimisis, 2013; Benitti & Spolaôr, 2017; Savard & Highfield, 2015). As discussed in our review discussing the educational potential in mathematics education, the positive effect of programming cannot be generalized, at least regarding improvement in the students' achievement in mathematics. The same trend is visible in studies only discussing robotics and mathematics education. For instance according to Lindh and Holgersson (2007) results regarding students' improvement in mathematics after training with Lego Mindstorms robots differed for different groups of students.

Furthermore, systematic use of formal mathematics is challenging with robots. Barak and Assal (2018) argue that the learning of formal mathematics and other STEM subjects can be challenging while using robots. According to Savard and Freiman (2016), even if students used mathematics in their robot-based activities, the use of mathematics was not systematic, as students did not design the use of mathematics in their problem-solving activities with robots. Instead, they started their problem solving with digital context, using a trial-and-error strategy above a systematic mathematical approach, which according to Savard and Freiman (2016), acted as an obstacle for greater mathematical understanding. The trial and error strategy is defined here as an iterative process where students test and repair the program without making any detailed plan until they succeed.

In summary, even if robot integration could provide out of school connections in a mathematics classroom and enhance comprehension and motivation, the links between mathematics education and robot-based activities remain unclear.

2.4 Mathematics and digital technology in the Norwegian school

The compulsory school system in Norway consists of a 10-year elementary school. Students begin their compulsory school in the year they turn six. The subject of mathematics has a central role in the national curriculum in Norway, where mathematics is seen as one of the main subjects, is part of cultural heritage and is seen as the basis of logical thinking. In the curriculum of mathematics, problem-solving is highlighted, and the use of digital technology is strongly present (Utdanningsdirektoratet, 2013).

The Norwegian school represents a suitable context for this study because Norway is planning to integrate programming within their mathematics curriculum (Utdanningsdirektoratet, 2018). Furthermore, the school system in Norway has a positive attitude towards the use of digital technology. In addition to the regular use of digital technology in regular education, students can choose technology as an elective subject (Utdanningsdirektoratet, 2013). Also, according to the survey about the use of ICT in education in Europe, Norwegian schools are highly digitally equipped (Wastiau et al., 2013).

In connection to the suggestion of curriculum reform in 2020 in Norway, programming is construed as a promising element of mathematics education: “Through programming, students can be more creative in approaching issues and gain the ability to explore connections that have not been possible to explore before.” (Utdanningsdirektoratet, 2019). The Norwegian school context is interesting for this study also due to the role of the teacher in programming integration. In Norway, 4,154 lower secondary teachers answered the 2018 TALIS (Teaching and Learning International Survey) conducted by the OECD (2019) in 48 different countries. The Norwegian teachers’ responses indicated a need for increased technological training for the teachers (Throndsen, Carlsten, & Björnsson, 2019). It is not enough to give the teachers new equipment for their classrooms; they also need advice as to how to appropriately integrate the digital technology in their teaching (Throndsen et al., 2019).

3. Theoretical framework

This study concentrates on a detailed analysis of the learning processes. As many of the earlier studies discuss the role of the teacher regarding integration of digital technology in mathematics classroom with the concept of instrumental orchestration or TPACK model, this study takes a broader and more detailed approach by discussing the role of the teacher in relation to other components in the classroom.

Ruthven (2014) compared the theoretical frameworks of TPACK model to instrumental orchestration in analyzing learning processes regarding technology integration in mathematics classrooms. He argued that these theoretical frameworks need to be supplemented with other theoretical components and ideas in order to be able to illuminate the findings in a more comprehensive and concrete manner. He highlighted the need to address the integration of digital technology with fuller and more systematic investigations; for instance, by synthesizing different theoretical components. The activity system analysis in CHAT is very suitable for this because it makes it possible to discuss the relationships between different components such as the role of the teacher, collaboration between students, the use of mathematical tools and the role of robots in the students' collective learning processes (Engeström, 1987).

The rationale of CHAT as a process theory is presented in the next subsection, which is followed by a short presentation of the history of CHAT. Furthermore, the components of the activity system analysis (see Figure 6) presented shortly in Table 2, are discussed in more detail in subsection 3.3, which is followed by the discussion about the basic features of CHAT. In the last subsection, I will discuss which parts of CHAT are more suitably this study, more deeply.

3.1 CHAT as a process theory

Engeström's (1987) CHAT is a suitable process theory for this study because it makes it possible to discuss learning and teaching processes in the classroom as intertwined activities by also discussing the gap between them. According to Engeström and Sannino (2012), many of the well-known process theories segregate learning and teaching as two separate processes without discussing the relationship between them. Engeström and Sannino (2012) argue that there is a gap concerning the relationship between learning and teaching processes. The teacher's intentions do not always fully meet with the students' real actions in the classroom. Thus, the discussion about learning and teaching activities by addressing different struggles, negotiations and occasional meeting points between them gives valuable information about real teaching and learning processes in the classroom. CHAT provides the possibility to discuss in detail the activities of the teacher and students, and the relationships between them, without assuming that they face each other (Engeström & Sannino, 2012).

The gap and relationship between learning and teaching processes are central for this study because, as discussed in our review, the role of the teacher is one vital point to consider regarding students' learning process (Article 1). The teacher's possible lack of knowledge about programming may complicate the learning and teaching processes in the classroom at the beginning of the integration. Thus, when discussing the role of mathematics in programming integration, both learning processes and teaching processes should be discussed at the same time, given that the teacher becomes a learner in their own right. There may arise situations where students know more than the teacher does. Thus, discussing learning and teaching as intertwined processes gives valuable information about real situations in the classroom when programming is introduced.

The traditional classroom model (see Figure 8) introduced in Chapter 2, where students and the teacher have two separate activity systems that meet in a common outcome, tests and grades do not suit this study. According to our review, learning processes with programming differ from traditional classroom activities with regards to the role collaborative learning and the teacher (Article 1). Thus, the model in Figure 8 cannot be used as such when discussing learning processes with robots. Firstly, as discussed in our review article in programming activities, students often work in groups. Secondly, the teacher cannot predict learning activities because programming activities are often problem-solving activities where the task development is constituted by the collective choices of the subjects of the activity (Article 1). Thus, the focus of the teacher cannot be on the given tasks in the same way as in a traditional classroom where the teacher gives a fixed task for a particular purpose for the students to solve.

And so, the activity system model (see Figure 6) provides the opportunity to discuss learning processes in the classroom by taking into account several different components concerning the whole process. For instance, the role of the teacher in students' learning processes can be discussed as a part of students' learning processes through the component of division of labor in the activity system model. The role of mathematics and the role of the robot can be discussed with the help of tools and objects. The collective learning processes can be discussed by positioning the group, which is learning collectively, as a subject in the activity system analysis. Engeström's activity theory characterizes the collective nature of the activities. In order to clarify this in more detail, I introduce a short history of CHAT in the following subsection.

3.2 The history of CHAT

Engeström's cultural-historical activity theory has its roots in the theories of Vygotsky and Leontiev (Engeström, 2005). Vygotsky's theory of cultural mediation is seen to be the first generation of activity theory (Engeström, 2005). Vygotsky (1978) introduced a triangular model of human action in the 1920s and early 1930s, where artifacts (tools) mediate the action between a subject and an object (see Figure 9) (Engeström, 2005). The subject is an individual who is aiming and relating to the object the goal of the action through mediating cultural artifacts, tools or signs (Yamagata-Lynch, 2010). The interaction with artifacts, such as different cultural tools, signs, and language, which again are results of social interactions, make collective meaning-making possible. Even if, according to Vygotsky (1978), the meaning-making process is social through different social artifacts, the actions are still carried out by the individual.

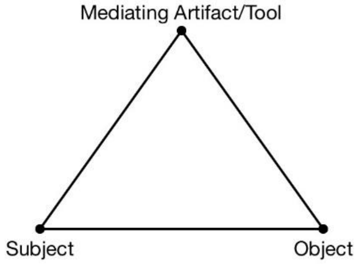

Figure 9. Vygotsky's model of mediated action, adapted from Engeström (2005, p. 60) and Yamagata-Lynch (2010, p. 17)

According to Engeström (2005), the turning point for the second generation of activity theory was when Leontiev developed Vygotsky's theory of cultural mediation further by adding a collective part through the separation of individual goal-directed actions and object-oriented collective activities. Object-oriented activity systems as unit of analysis consisting of individual actions enabled the analysis of collective meaning-making processes (Yamagata-Lynch, 2010). However, the object of activity was mostly presented by Leontiev as a theoretical tool without practical implications (Kaptelinin & Miettinen, 2005).

Engeström (2005) argued that Leontiev did not present his suggestion about a collective activity system as a graphical model. Engeström (1987) further developed an activity system analysis model (see Figure 6) from Vygotsky's original model of mediated actions and Leontiev's suggestions about collective object-oriented activities. He added the collective components: rules, community, and division of labor, in the original Vygotskian model (Engeström, 2005). The collective components and relationships between different components in the activity system model enable the analysis of interactions between individuals and groups in meaning-making processes (Yamagata-Lynch, 2010). As tools mediate relationships between subjects and object, rules mediate the relationship between subjects and community and division of labor mediates the relationship between community and object (Cole & Engeström, 1993).

In the following subsection, I introduce each component in the activity system analysis and the potential role of them as mediators in robot-based activities.

3.3 Components in the activity system analysis

3.3.1 Subject of the activity

The subject of the activity can be an individual or a group. Mediated actions in Vygotsky (1978) are individual, and the subject of action is an individual who is related to the object through social tools. In CHAT, activities are always collective, and the subject of an activity can be an individual or a group who is concerning the same object (Engeström, 2008).

Joint activities can, however, consist of separate individual actions. The individuals of the collective activity can relate to the same object in different ways. Thus, individuals can aim towards the same object through separate individual actions in the same collective activity (Bødker, 1996; Kaptelin & Nardi, 1997).

As discussed in the review article, students often worked in groups during programming activities (Article 1). Thus, when discussing programming activities with robots, it is natural that the subject of the activity is a group of students. When working in groups, members can take different roles. One can, for instance, be a programmer, while another is conducting some calculations, measurements or tests. The roles can also be changed. As a result of this, the activity consists of the individual actions of subjects, which all are related to the same collective object.

3.3.2 Object of the activity

Subjects of the activity work collectively towards their common object. The object is the foundation for the whole activity and a very central concept in the activity theory (Kaptelinin, 2005). The object answers the question: *why is the activity taking place?* It is the ultimate reason for the activity (Kaptelinin, 2005). Engeström (2005) is from Finland and in order to describe the object, he refers to the Finnish folklore Kalevala, which was gathered from stories in Finland and first published in 1835. Here, the ultimate object Sampo, which was made by the mighty primeval smith, was the source of wellbeing and wealth and was the source of power struggles. The object is something more than its physical form; it awakes passion and anger, it is whole or part, invisible or highlighted, and it gives endless possibilities. This is, in Engeström's interpretation, the crystallized definition of an object in any productive activity.

According to my interpretation, Sampo can be seen as the source of the wellbeing of its time. In Kalevala, Sampo brought currency, grain, and salt (Lönnrot, 1849). In Finnish culture like in any other culture, there have been different Sampo-type objects that have been crucial in building the nation's wealth and social structures. Before industrialization, it was the saw and ax which enabled large-scale forestry, tar production and later paper products to be exported. In other eras, it was perhaps the cellphone brand Nokia which presented Finland on the world stage as a technology innovator. Today, Sampo could be, for instance, worldwide technology development and digitalization, and their mastery.

The group, where the members are sharing the activity, often has the same collective object, which can be material or nonmaterial, like a plan or common idea (Kuutti, 1996). Even if the collective object is the motivating and steering factor in the activity, it is not directly the motive of the activity (Kaptelinin, 2005). According to Kaptelinin (2005) this is one factor that differs between Leontiev's and Engeström's approaches in activity theory. Leontev (1978) connected the concept object directly with the concepts of motive and needs. According to him, an object of an activity is "a true motive of an activity." However, Engeström (1987) related the concept of the object to collective production. As activities consist of several individual actions with individual motives, there can be several different motives behind the object (Kaptelinin, 2005; Nardi, 2005). Thus, subjects of the activity are not always aware of the motives behind the collective activity, and objects of activities are complex systems (Miettinen, 2005).

A collective object formation is a design process where the subject of the activity negotiates a common object, which takes form, transforms and possibly changes during the activity development (Engeström, 2008; Kaptelinin, 2005; Miettinen, 2005). During design, forming and transformation processes, the history and actions of different subjects influence object development. Thus, an object and its transformative processes are complicated with different layers and nuances (Engeström, 2008). Due to the manipulations of subjects, activities develop, and transform.

The collective transformative processes may lead to expansion of the object, which leads to expansive learning. In CHAT, learning is seen as a collective and an expansive process unlike traditional learning theory, which sees learning as an individual change:

> Traditionally we expect that learning is manifested as changes in the subject, i.e., in the behavior and cognition of the learners. Expansive learning is manifested primarily as changes in the object of the collective activity. In successful expansive learning, this eventually leads to a qualitative transformation of all components of the activity system. (Engeström & Sannino, 2010, p. 8)

Engeström's view of learning is broader than the traditional learning approach. Traditional learning theories view learning as a change in an individual's mind, behavior, or social actions (Cobb, 1994). In CHAT, learning manifests as a multidimensional development, transformation, or change in the components of the activity system. Thus, knowledge is distributed between different participants and components in the activity system analysis. The learning is not just a one-dimensional individual change.

In a traditional classroom (see Figure 8) where students aim towards the given object individually, learning is measured with individual tests, and learning is seen as an observable change in a subject's knowledge or skills. Furthermore, students and their learning are objects in the teacher's activity system model in a traditional classroom setting (see Figure 8). As mentioned, student and teacher activity system models meet in a shared desired outcome, test results, and grades. The knowledge and skills sought are predictable and well-defined, as the teacher knows in advance what is to be learned (Engeström, 2005). Engeström (2005) viewed learning in a broader perspective; according to him, learning is not always defined beforehand, stable, or predictable. Engeström discussed learning in organizations and argued that people are also learning many things other than just the predictable or planned skills and knowledge in organizations and in their personal lives. In a classroom context, Engeström (2005) used the term "hidden curriculum" to expose the other knowledge and skills that students learn, in addition to the planned or predicted learning. The "hidden curriculum" can, for instance, be knowledge about, "how to please the teacher, how to pass exams, how to belong to groups" (Engeström, 2005, p. 66). This kind of informal learning is always present in the classroom along with planned ordinary learning, and it may be in contradiction to formal learning. Engeström (2005) described learning in a broader manner by discussing expansive transformations that evolve when participants involved in the collective activities begin to question and deviate from existing norms and make elaborate efforts toward collective changes in the activity.

The expansion of the object can manifest in several different dimensions; for instance, by considering who is included, what the future visions are, or how the activity has enriched (Engeström & Sannino, 2010). According to my interpretation, this view of learning has potential to bring more nuances into the learning perspectives used in a classroom assessment, which enables us to consider learning in a broader view by bringing more dimensions to the operationalization of learning and by giving the possibility to view what actually happens in learning processes in a classroom; this, in turn, presents the possibility to assess collective learning processes, instead of individual results, by discussing, for instance, who is included in the activity, how the activity is enriched, what the scope of the object is, in which direction the activity is developing, or what the future views are. It is notable, however, that there may also arise unexpected changes in the activities (Engeström & Sannino, 2010).

In this study, I approach learning as a collective process by using Engeström's view of learning, which enables me to discuss how different components, such as the role of the teacher, collaboration between students, the role of mathematics, or the role of robots, influence students' learning processes. I discuss the students' collective activities in the classroom by concentrating on how different components and relationships between them develop and transform. I view, for instance, the manner in which the object of the activity is negotiated, what the role of the teacher during the object negotiation phase is, how the object of the activity relates to the tools in use, the ways in which the tools in use or other components in the activity system influence the object development, and how the object evolves during the activity development. Analysis of collective negotiations between students and between students and the teacher provides information about the object of the activity during activity design and development; it also provided information about who is included in the activity. Thus, the activity system analysis provides a broad-but-nuanced view of learning in this study.

The approach to view learning as a collective process is complex to analyze, and it has limitations. One such limitation is that determining the collective object of the activity for any given moment during the activity development as the object of the activity is a complex, dynamical and multilayered component. The other limitation is that learning in CHAT is not always a positive change or transformation. However, as this approach provides possibilities for a nuanced analysis of students' learning processes, I considered this approach as being suitable for this study, despite these limitations, and I try to take these limitations into account in my analysis. Furthermore, I sought to find potential transformations in classroom activities and potential links between programming activities and mathematics. As I intended to discuss learning processes activated and what actually happens in the classroom, I did not attempt to determine positive or negative learning results. The analysis process and the complexity to determine the object of the activity is discussed in greater detail in the Methodology section.

In a classroom context, there are boundaries between the students' classroom activities and their everyday activities outside of the classroom (Akkerman & Bakker, 2011), and boundaries between the teacher's and students' activities (Engeström, 2008). As discussed, in a traditional classroom, where students are given predefined tasks to solve, students have difficulties making connections between activities in the classroom and their lives outside of the classroom (Boaler, 2009; Bray & Tangney, 2017; Hoyles, 2016). School mathematics differs from mathematics outside of the classroom, and teachers and students do not necessarily share the same intentions, motives, or objects. However, according to Engeström (2008) the traditional classroom model is complicated to change, because of its robust structure. He provides suggestions based on CHAT for changes on a general level in the traditional classroom. There is need for boundary-crossing in order to make changes in a robust traditional classroom model (Engeström, 2008). In that change, the object of the activity has a central role. He suggests that instead of manipulating or changing some single components in the activity system model, there needs to be changes in objects with new motives in order to cross boundaries between different activities. This is a theoretical claim for the reflection of which Engeström (2008, p. 90) offered concrete questions to consider: "What could replace the text as the object of schoolwork? And how could such a transformation take place in practice?" Engeström (2008) claimed that the transformed object of education is accessible through comprehensive transformations of all the components on the activity's system model by breaking and transforming the robust structure of the traditional classroom model.

Engeström (2008) suggested in his later work that the two different activity systems could interact with each other by totally or partly sharing the same object: a boundary object (see Figure 10). The concept of boundary object between intersecting social worlds was originally defined by Star and Griesemer (1989, p. 393) as follows:

> Boundary objects are objects which are both plastic enough to adapt to local needs and the constraints of the several parties employing them, yet robust enough to maintain a common identity across sites…They have different meanings in different social worlds but their structure is common enough to more than one world to make them recognizable, a means of translation.

Based on this definition, two different worlds (i.e., activities in CHAT) can at the same time communicate and increase autonomy of these separate worlds through a boundary object, even if the boundary object can have different meanings in each of them (Trompette & Vinck, 2009). Even if these different worlds (i.e., activities) may not be in a consensus, they can still collaborate (Star, 2010).

The theory of boundary objects in Star and Griesemer (1989) stem from Actor Network Theory (ANT) (e.g., Latour, 2005), which does not distinguish subject and object as CHAT does (Engeström & Escalante, 1995; Miettinen, 1999). While CHAT discusses the dialectic relationship between subject and object, ANT is based on symmetry between subject and object (Miettinen, 1999), and thus, human and non-human actors have the same role in ANT (Engeström & Escalante, 1995). However, even if CHAT and ANT have differences, the concept of mediation has a central role in both of the theories (Miettinen, 1999). ANT is based on symmetrical mediation between human and non-human actors, and knowledge and skills are distributed among people and tools (Miettinen, 1999). Engeström (2005) saw ANT as one foundation to the theory development of interactive activity systems in CHAT, and he called the development of interacting activity systems the third generation of activity theory.

Figure 10. Two interacting activity systems with boundary object, reconstructed from Engeström (2005, p. 63).

The concept of the boundary object could be used, for instance, by connecting classroom activities with the activities outside of the classroom. The robot could interact as a boundary object between students' activities inside and outside of the classroom. The *school-going* activity system could interact with the activity systems outside of the classroom. However, because this study concentrates on classroom activities without paying attention to, for instance, students' outside-of-school activities or communications, it is not possible to discuss students' boundary objects between classroom activities and out of school activities in this study. As the aim of this study is to discuss the potential links between mathematics and student programming activities in the classroom, the focus is on classroom activities, and the scope of this study is not exceeded on students' outside-of-the-classroom activities. Even though the boundaries between students' school-going activities and the activities outside of the classroom is an interesting research theme, it would require a different study design with a different kind of approach and research questions.

Furthermore, in a classroom context, students' activity system and the teacher's activity system could interact with each other through a boundary object. However, according to my interpretation, the boundary system model is not optimal in this study because the model isolates the teacher's activity system far from the students' activity systems.

Boundary objects reduce the need for communication between interacting activity systems because the interaction is possible through boundary objects. The model of boundary objects is in earlier studies criticized because of the limited communication between interactive activity systems (Akkerman & Bakker, 2011). As Figure 10 shows, the activity systems only meet through boundary objects.

Otherwise, activities are separate, and the other components in the activity systems are not connected. I conclude that the boundary object model is not suitable when discussing intertwined teaching and learning processes at the beginning of the robot integration. As mentioned in the introduction section, at the beginning of the robot and programming integration, there is a possibility that both students and the teacher are novices in programming. Thus, the boundaries between learning and teaching processes can be blurred, which highlights the importance of communication between students and the teacher during activity development. Therefore, in this study, there is a need for discussion about broader interaction between students and the teacher's activity systems by also discussing the potential for other meeting points between these activity systems other than in activity systems with boundary objects.

Thus, I will focus on collective group activities and communication between students and the teacher in the classroom by using Engeström's earlier work considering separate activity systems in the second generation of activity theory. In the following subsections, I will discuss the other components in the activity system model.

3.3.3 Tools in the activity

Tools mediate and shape the activity between the subject and the object. Activities are always mediated by tools, which can be working tools, for instance, robots and computers, or non-material such as knowledge and skills (Engeström, 2005). In this study, the tools in use are, for instance, computers; robots; programming language and skills; or, different mathematical tools.

Tools are developed as a result of collective activities. For instance, mathematics is a cultural tool, which is a result of human beings' collective activities over time (Engeström, 2005). Thus, activity development is constituted by the history of different tools. For instance, in robot-based activities, different participants have their histories concerning their use of programming tools. If the mathematics teacher is a novice in programming, the situation might be that some students may have more significant programming skills than the teacher. The situation might be reversed with the mathematical tools that are used.

The use of tools is often unconscious. However, at times, when the tool is not working as desired, the focus can be on tools, and tools may even become the object of the activity (Engeström, 1987). According to Engeström (1996), tools and object have a dialectical role. Even if tools may become objects of the activity and vice versa, they are two very different components in the activity system with different roles. My interpretation of this is that in robot-based classroom activities, the object of activity can be to program the robot to act as desired. The mediating tools in that activity can be different programming tools such as a programming language. And so, these tools are not the object of the activity. Programming language as a tool has a very different role in the activity than the object, which is the goal, drive, and direction aimed for by students. The focus can temporarily be on a programming language, or it may become the object of the activity when making errors in programming. Then the role of the programming language changes in the activity system from tool to object. When the problem is solved, i.e. object is reached, the programming language can become a tool again.

Furthermore, even if a collective activity consists of individual actions, collective activity is not a straightforward sum of these actions (Engeström, 1996). According to my interpretation, individuals who are attending the activity with different actions aim towards the same object probably by using different tools, partly simultaneously and together, partly individually. For instance, in robot-based activities, students can divide a programming task into smaller parts by using different tools, individually or together. Even if students work partially individually, they are interacting with each other at least with some cultural tools such as gestures and words. Furthermore, the classroom rules and other students in the classroom influence the students' collective activities.

The three components in the bottom part of the activity system: rules, community and division of labor, greatly influence the development of collective activity by complicating its straightforwardness as a sum of individual actions (Engeström, 1996). I will discuss these components in more detail in their respective subsections below.

3.3.4 Division of labor

As previously mentioned, activities are multi-voiced, multi-motivated, and multilayered because of the different viewpoints and histories of the subjects. The voices of subjects will be heard through division of labor (Engeström, 2008).

According to Engeström (2008) behind the division of labor in a classroom is hidden boundaries that have influence on activity development. In a traditional mathematics classroom, the boundaries can be related to the role of the teacher, where the teacher is the absolute authority, and the role of the teacher is to be the arbiter of knowledge (Bray & Tangney, 2017).

In this study, I use the concept of division of labor in order to discuss collaboration between students and the role of the teacher in the classroom when robots are integrated. According to Engeström (2008), a new element in the classroom, such as new technology, has the potential to cause contradictions in components of the activity system. Concerning the division of labor, some old elements, such as the role of the teacher, may collide with the new activity (Engeström, 2008). This potential is present in programming and robot integration because the situation may be that some of the students have more knowledge about some parts of the new technology. This collides with the traditional classroom model, but at the same time gives potential for new activity development and boundary crossing in the classroom.

I will introduce the concept of rules as one component of the activity system in the following subsection.

3.3.5 Rules

Rules are one of the collective components at the bottom of the activity system triangle, which have a significant influence on the development of collective activity through other components in the activity system. Rules are the regulations that formally or informally affect activity formation and development (Yamagata-Lynch, 2010). Rules can be visible or hidden. Visible rules in the classroom can be, for instance, task assignments or time limitations. In addition to these, there are several hidden rules to consider in a traditional classroom.

Traditional mathematics classrooms have quite rigid rules which influence even outside of the mathematics classroom. According to Engeström (2008) rules can act as boundaries for activity development. Specific taken-for-granted rules from traditional classrooms can act as hidden boundaries in reformation of classroom activities. These boundaries can include testing and grading practices; the use of textbooks; the use of time; outside of classroom connections, and different patterns regarding student grouping; control; discipline; and, interactions between teachers and students. According to Engeström, these hidden boundaries must be made visible and questioned in order to make changes in the classrooms and the objects of education, in this case in mathematics education.

In addition to the classroom rules, the use of mathematical tools has specific rules. As discussed, in traditional formal mathematics education, students connect their learning with abstract rules and memorization when using different formulas or algorithms, as an example (Albert & Kim, 2013; Bray & Tangney, 2017; Hoyles, 2016, 2018; Opheim & Simensen, 2017; Pietsch, 2009. These rules may influence activity development when mathematical tools are in use.

So, rules are connected to the whole community, in this case, to the whole classroom. I will briefly introduce the concept of community in the activity system model.

3.3.6 Community

Community is a social group where subjects of the activity belong during the activity development (Yamagata-Lynch, 2010). In the classroom activities, the community is all the students, the teacher, and other adults or persons in the classroom. Members of the community can be part of different activity systems in the same classroom. Subjects of different activity systems may interact with subjects of other activity systems. Some of the subjects, such as the teacher, may even be a part of several activities at the same time. Thus, activity systems are not isolated; other activity systems throughout the community influence them.

3.3.7 Outcome

The outcome is the result of the activity (Yamagata-Lynch, 2010). As seen in Figure 8, in a traditional classroom model, teachers’ and students’ activity system models meet only in the desired outcome: tests and grades. In this study, the focus is on learning processes instead of on outcomes. The outcomes of the activities in this study can be that students get robots to act as desired or students learn some programming or mathematics, collectively. As mentioned, learning in this study is discussed as a change in the collective object instead of as an individual outcome at the end of the activity.

I will discuss the characters of CHAT in more detail in the following subsection.

3.4 The character of CHAT

As discussed, CHAT is a proper tool to analyze collective learning processes in the classroom. To build this tool, Engeström summarized CHAT with the following five principles: 1) The activity system analysis is a prime unit of analysis; 2) Multi-voicedness of the activities; 3) Historicity of the activities; 4) Contradictions within or between different activity system models; and 5) The possibility for expansive transformations. Engeström (2005) saw the activity as collective, tool-mediated and object-oriented. He analyzed activity against its history. In order to build the tool for the analysis in this study, these principles, as mentioned above, will be discussed and summarized in more detail in the following subsections.

3.4.1 Activity system analysis

According to the first principle, Engeström's (1987) activity system (Figure 6) is the prime unit of analysis in CHAT. As mentioned in the beginning of this chapter, Engeström's (1987) activity system analysis makes possible the analysis of the role of the teacher and the role of mathematical tools during the students' whole collective learning process. The activity system analysis in CHAT considers the tool-mediated activities against the collective components in the activity system (see Figure 6). The seven components, subject, object, tool, rules, community, division of labor, and outcome (see Table 2), are in relation with each other. And so, because of the collective nature of the activities, the relationships between components in the activity systems have diverse layers. The activities being multi-voiced are discussed in the following sub-section.

3.4.2 Multi-voicedness

CHAT is based on an interest in collaborative activities. The subjects of the activity carry their histories, which affects the activity through the division of labor. There are always multiple points of view, traditions, and interests behind the activity (Engeström, 2005).

Programming tasks are solvable in several different ways. Thus, during the collaborative problem-solving activities with robots, there are several opinions among the subjects. These opinions and ideas develop the activity through the division of labor. Therefore, activity development is not predictable. Even if the teacher designs the tasks, they do not know beforehand what kind of problem-solving activities will be activated. That may change the role of the teacher in the classroom because the teacher does not know what is to be learned beforehand. Activities develop due to interactions and negotiations between subjects, which all have their background as well similar to the other components of the activity system. Thus, history is an integral part of activity development. This is discussed in the following subsection.

3.4.3 Historicity

According to Engeström (2005), activities shape and transform over time, and history should be a part of the analysis. The activity development is constituted by the history of different components in the activity system such as tools, objects, and rules, which mediate and shape the activity. Besides, the subjects of the activity have their history concerning these components (Engeström, 2005).

Historicity is strongly present in classroom activities. Classroom activities are carried out with different kinds of historical rules and social practices. In the traditional classroom model (see Figure 8) which has established practices, the historical rules and social practices play a significant role. Certain practices with specific rules are present in classroom activities because of the strong history they possess. These practices relate to homework activities, teaching activities, the culture of how to behave in the classroom and evaluation practices. These practices include fixed tools in use such as blackboard, chalk, pen, and textbook, due to their long history.

Conclusively, even if new kinds of innovative learning processes take place in the classrooms, the historicity of the practices of the traditional classroom model may be present in the classroom activities. This may, for instance, be related to the role of the teacher.

Programming and robots may have the potential to change the role of the teacher in the classroom. Regarding the programming tools, the students and the teacher may have a different history. Outside of the classroom, students may participate in different technological and digital activities than the teacher does. Students bring the historicity of these activities with them to the collective activities in the classroom, as the teacher brings their historicity of pedagogical activities with them.

Thus, students and teachers can attend collective activities with different kinds of ideas and skills.

Different histories of the subjects and activities may change the division of labor in the classroom and cause tensions or contradictions, in the components or between the components, in the activity system analysis. Tensions and contradictions are an essential part of activity development. I will discuss the potential of contradictions for activity transformations in the following subsection.

3.4.4 Contradictions and expansive transformations

During the activity development, different tensions and contradictions within or between different activity systems may arise (Engeström, 2005). Engeström (2008) sees historically accumulated tensions and contradictions as a potential for innovative changes and transformations in activities. Contradictions also help in understanding the sources of troubles in different kinds of teamwork. Engeström (2008) discusses teamwork in different project teams in different work organizations. Regarding this, he discusses knowledge- and innovation-driven production, where contradictions may arise between pressure for economic growth, pressure of mass production and quality assurance with innovative development.

When discussing robot integration with Engeström's (2008) ideas about contradictions in knowledge- and innovation-driven activities, the activities in the classroom with robots can be in contradiction with pressure coming from outside of the classroom on a large scale. As discussed in the introduction, programming integration relates to 21st-century skills, which are essential for future society. In the 21st century skills, development of innovative and creative ideas is highlighted, in addition to technological knowledge. However, in today's society, which is increasingly competitive with high pressure of economic growth, education is also increasingly competitive. Economic growth is depending on education of digital technology and education in general, where individual, national or international tests measure competitiveness. That puts pressure on national education designers, curriculum developers, teachers, and students. So, the pressure from these indicators is in contradiction with the goals of creative and innovative teaching and learning environments.

On the other hand, contradictions can enable potential positive changes and transformations in activities, as they may be the source of collective learning. As discussed earlier, learning in CHAT is seen as a collective, expansive process affected by different components in the activity system due to the system's historicity and contradictions. Activity development and contradictions within it may escalate into significant changes in the object of the activity. The whole object may be transformed and expanded due the contradictions in the activity system. Thus, the activity develops towards a new more extensive collective object (Engeström, 2005). A new more extensive object provides a broader opportunity with a broader horizon of possibilities than with the previous activity to learn *something that is not yet there* (Engeström & Sannino, 2010, p. 2).

The components of activity system analysis and these five principles discussed above are the critical elements of the theory used in this study. In the following section, I will describe how these elements are used in the different articles of this study.

3.5 Theory use in the articles

In order to answer the overall research question, it has been divided into three smaller questions based on the components in CHAT. These questions are answered with the help of operationalized tools from CHAT. In this section, I will discuss how I have operationalized the theory used in each article.

3.5.1 Article 1

As mentioned, the first article is a literature review article, where Odd Tore Kaufmann, and I answered the question: *What is the educational potential of programming in mathematics education?*

As Article 1 was a literature review, it did not include a comprehensive theoretical framework. However, Article 1 acted as an initiator for the use of CHAT in this study. As discussed, earlier studies concentrated mostly on the learning results instead of learning processes. Thus, the need to use a process theory in this study was apparent.

Furthermore, as mentioned, the collaboration between students and the changed role of the teacher was characteristic for programming activities. However, the effect of these components was not widely discussed in earlier studies. The activity system analysis in CHAT enables to discuss these components as a part of the students' learning processes. Thus, choosing CHAT as a theoretical framework in this study seemed natural. The approach of the second and third articles is based firmly on the activity system analysis in CHAT. I will discuss the use of CHAT in these articles in the following subsections.

3.5.2 Article 2

In Article 2, Geir Afdal and I concentrated on the use of mathematical tools in robot-based activities by answering the question: *What is the relationship between mathematical tools and object in robot-based collective student learning activities in secondary education?* In order to answer that question, we used activity system analysis in CHAT. The main focus was on the tools and the objects of students' collective activities.

First, we aimed to get a broad understanding of activity development. In order to do that, we used the key concepts from activity system analysis. We coded our transcribed data material with the concepts of subject, object, tools, rules, community, and division of labor. After that, we focused mostly on the use of mathematical tools and object development, because the tools in use is constituted by this. The more in-depth focus was placed on the relationships between different components, particularly on the relationship between tools in use and object development. The changes in the components and the development of the relationships between the components over time were analyzed.

Furthermore, we paid attention to the multi-voicedness and historicity of activities. The possibility of expansive transformation was also part of our discussion. In order to understand multi-voicedness, we concentrated on interactions between subjects and between subjects and tools. The historicity of the activity was addressed by analyzing the activity development. Expansive transformations became visible through the analysis of object development.

3.5.3 Article 3

In the third article, I answered the question: *How does the role of the teacher in robot-based activities influence students' learning processes in mathematics?* Again, the activity system analysis, with the seven components, was the unit of analysis. The focus was on the division of labor because the role of the teacher was discussed through that component.

And so, *how the role of the teacher influenced the activity development* became the main focal point of analysis, by considering the historicity and the multi-voicedness of activities. Again, I started by coding the data material with the components from activity system analysis, subject, object, tools, rules, community, and division of labor. After that, I discussed the relationship between the role of the teacher as a part of division of labor and other components in activity system analysis. The analysis was hence focused on the relationship between division of labor and tools, and the relationship between division of labor and the object development.

4. Methodology

In this chapter, I will present and discuss my methodological choices. In order to illustrate the relationships between the components in my study design, I use Maxwells's (2005, p. 217) Interactive Model of Research Design (see Figure 11). In this study, each component in the model: goals, research questions, conceptual framework, methods and validity of the study, are harmoniously and dynamically connected, as Maxwell (2005) had suggested. Thus, the study design process is not a simple plan; it is a real entity within the study.

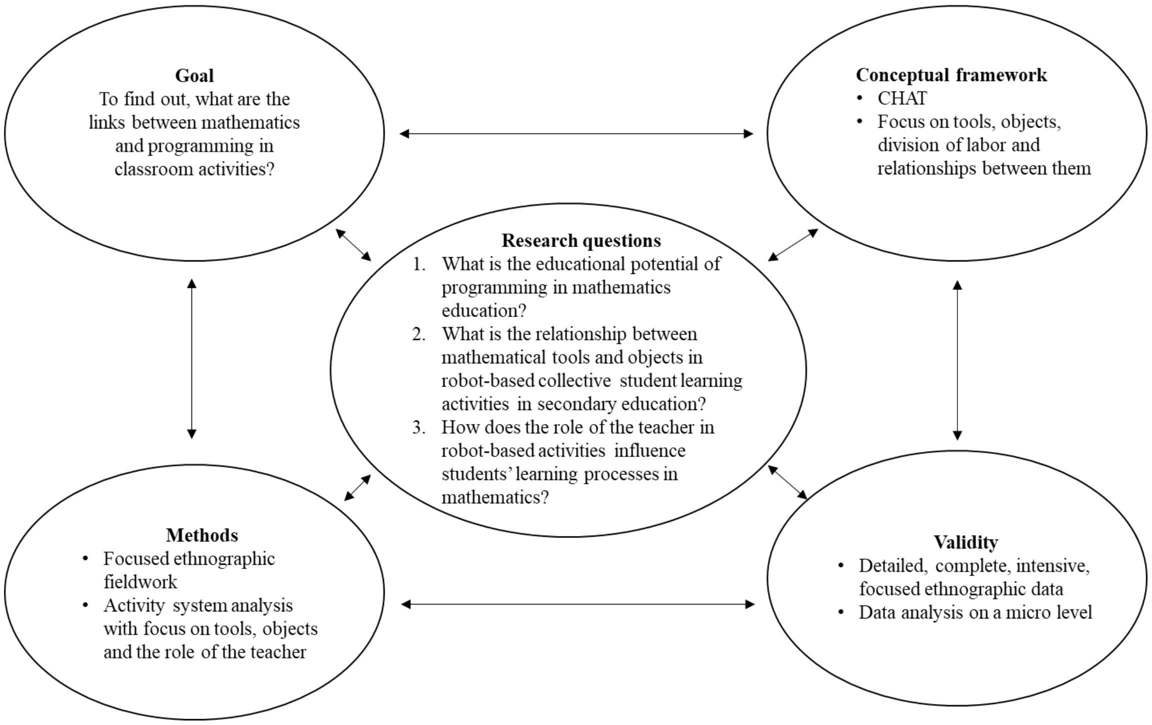

Figure 11. Maxwells's (2005, p. 217) Interactive Model of Research Design, modified for my study

The upper triangle in Maxwell's (2005) model, goals, research questions, and theoretical framework, are discussed and justified in the introduction and theoretical framework sections of this thesis. The components, methods, and validity connected with the research questions are discussed in this methodology section. In addition to that, the interaction between theoretical framework and methodological choices and validity of this study are discussed. As ethics are part of each component in the study design, the ethics of this study are discussed in the last subsection of this methodology section.

In the next subsection, I will introduce the research strategy of this study, a focused ethnography, and how it relates to other components in the study design. After that, I will discuss how CHAT relates to the methodology of this study. Then I will present the whole data collection and data analysis methods in this study. Also, my role as a researcher is discussed. In the last part, I will discuss the quality of this study with the concepts of reliability, validity, and ethics. I will reflect these concepts on other components in the study design.

4.1 Focused ethnographic research strategy

A focused ethnographic research strategy is suitable to get an understanding of the activities in the classroom with robots. In the ethnographic studies, the researchers aim to deeply understand human groups, their activities, and culture by sharing the same social space with the informants (Madden, 2017). However, a focused ethnography differs from traditional ethnography by number of features, many of which are relevant to this study. The central features of the focused ethnography, which are relevant for this study are listed in Table 3. In the following, I will discuss the relevance of these features for this study.

A focused ethnography is suitable for this study because the focus of the study is on classroom activities with robots (which take place in specific lessons during a school week), and so short-term field visits are suitable. Ethnographical studies have a difference in scope, depending on social units studied and the duration of the data collection (see Figure 12). The broadest studies, with data collection of many years, involving several researchers, belong in macro-ethnography. Micro-ethnographies, where studying is focused on unique social situations, require much less time (Spradley, 1980). In a focused ethnography, also called a short-term ethnography (Pink & Morgan, 2013), fieldwork is significantly more time-intensive than in a traditional ethnography (Knoblauch, 2005; Pink & Morgan, 2013; Skårås, 2018). In this study, concentration was placed on everyday activities on a micro-level in order to get a detailed understanding of the collective learning processes that are activated, and the links between mathematics and the activated learning processes. This study is closer to a micro-ethnography, given that observations were made of multiple social situations in the classroom, as the collective classroom activities include several social situations. The observation of multiple social situations on a micro-level provided detailed and comprehensive enough data to understand the activities in the classroom. Thus, there was no need for macro-ethnography, which is needed more in the studies of complex societies or multiple communities. The long-lasting familiarization and immersion with participants are needed when a researcher is jumping in an unfamiliar environment and when the aim is to deeply understand the practices of human groups (Skårås, 2018). However, when the research field, such as a classroom in this study, is familiar to the observer, shorter fieldwork can be an option (Bernard, 2006, p. 349; Skårås, 2018), because there is no need for a long familiarization process.

Scope of research	**Social units studied**
Macro-Ethnography	Complex society
	Multiple communities
↕	A single community study
	Multiple social institutions
	A single social institution
	Multiple Social situations
Micro-Ethnography	A single social situation

Figure 12. Different ethnographical research scopes, retrieved from Spradley (1980, p. 30)

Secondly, in a focused ethnography, short-term fieldwork is compensated with more intensive data gathering methods, such as audiovisual recordings (Knoblauch, 2005; Skårås, 2018). As in a traditional ethnography, researchers write field notes; but, in focused ethnography, audiovisual recordings are more common (Knoblauch, 2005). Audiovisual recordings enable a detailed and intensive analysis of the activities in the classroom. With the help of audiovisual recordings, a detailed analysis of interactions between students and the teacher with different kinds of gestures, communication, and tools in use is possible on a micro-level. With audiovisual recordings, the possibility to go back in the data repeatedly exists, allowing writing more detailed field notes after the fieldwork is complete. It is also possible to make transcripts of audiovisual records and use systematic coding of data material afterward.

Thirdly, the focus in a focused ethnography is on separate key informants instead of the whole community (Knoblauch, 2005; Skårås, 2018). As mentioned in our review article, robot and programming activities are often collaborative, and the activity development depends on the collective choices that students make during their problem-solving activities (see Article 1). Thus, it is impossible to observe each group and their work in detail. Therefore, for this study, it was suitable to focus on one group of students and their activities in the classroom by having them and the teacher as key informants.

Fourthly, in a traditional ethnography, a researcher participates in activities with informants, but in a focused ethnography, a researcher acts more like an observer (Knoblauch, 2005; Skårås, 2018). This is an essential point in this study because I was interested in getting a natural picture of the learning processes in the classroom where robots are introduced. For example, my participation in teaching as a researcher would change this natural situation considerably. Thus, I participated only in the beginning by introducing robots for the teacher shortly; otherwise, my role was closer to that of an observer. In greater detail, my role was as a moderate participant, which is between an active and a passive participant, as I in the beginning participated by introducing Lego Mindstorm robots to the teacher, after which I concentrated more on classroom observations. I will discuss my role in the classroom in further detail in Section 4.3, where I present the data collection methods in this study.

Table 3. Differences between a traditional ethnography and a focused ethnography adapted from Knoblauch (2005, p. 4).

Conventional ethnography	**Focused ethnography**
Long-term field visits	Short-term field visits
Experientially intensive	Data/analysis intensity
Time extensity	Time-intensity
Writing	Recording
Solitary data collection and analysis	Data session groups
Open	Focused
Social fields	Communicative activities
Participant role	Field- observer role
Notes	Notes and transcripts

As I have mentioned at the beginning of this chapter and within the theoretical framework of this study, CHAT has a strong relationship with my data collection and analysis methods. In the following subsection, I will discuss how different components in CHAT affected my data collection and analysis. Following this, I will conclude the discussions of this subsection regarding focused ethnography to continue onto the next subsection about the influence of CHAT by introducing my data gathering and data analysis methods in detail.

4.2 The five principles in CHAT

The five principles of CHAT introduced in the theoretical framework section describes the usefulness of CHAT. These five components affected the data gathering and analysis methods in the study, as well. CHAT affected different components in the study design, such as the unit of analysis and methods, as well as the epistemological aspects of the study. Thus, learning processes with robots are discussed in this study through the developments of collective activities with particular focus on the use of tools, object development and division of labor during different phases in the activity development.

4.2.1 Activity system analysis

The focus, in my study, is on the teaching and learning of mathematics, while the aim is to investigate: *what are the links between mathematics and programming in* *classroom activities?* As previously discussed, in the introduction section and the theoretical framework, the role of mathematics in collective programming activities can be discussed holistically with the activity system analysis in CHAT. In this study, the unit of analysis is a collective classroom activity as a whole, and *not* individual subjects or other parts of the activity system. Matusov (2007) argues that because of the complexity of collective human activities, social activities should be analyzed by addressing all activity components holistically, and *not* by having only individuals or individual properties as a unit of analysis. Individuals are often part of a more complex activity system, where several components influence activity development. In this study, I concentrated on students' collective activities in the classroom. All the components in the activity system model, such as: the group of students as a subject of the activity; robots as a tool or object of the activity; the role of mathematics as tool and object of the activity; and the role of the teacher through division of labor, are discussed holistically as part of my analysis.

The activity system analysis with the different components as a unit of analysis, determined how I handled the data material after the fieldwork. I will explain my coding and analysis methods later in this chapter.

4.2.2 Multi-voicedness

From the perspective of CHAT, the activities are collective and multi-voiced. As discussed earlier in robot-based activities, the activity development is constituted by the collective choices of the group, which are constituted by many different ideas and viewpoints among subjects of the activity. The interactions between the subjects of activity and other participants are visible through different tools and methods of communication in use, which can also be nonverbal, thus involving gestures, smiles, laughter, or grimaces.

The multi-voicedness of activities affected observations in the classroom. In my observations through audiovisual recordings, I concentrated on interactions, collaboration, and communications between my key informants. The focus was on different gestures, expressions, and actions, such as the nonverbal cues expressed above, between students and the teacher. This is natural for ethnographical study, which is describing human behavior in a versatile way in the subject's cultural environment (Lappalainen, Hynninen, Kankkunen, & Lahelma, 2007).

4.2.3 Historicity

As discussed in the theoretical framework section, according to the principle of historicity in CHAT, historicity should be part of the analysis in all activities (Engeström, 1987). In this study, historicity is present in several components. The classroom, as such has long historicity, which influences activities in the classroom. Furthermore, mathematics education has long historicity with certain associated expectations and rules.

This study is a continuation of existing classroom activities by adding new components, such as robots and programming, which influence the short time historicity of the classroom activities. As the historicity is a part of activity system analysis in CHAT, the duration of the data gathering must be long enough. Thus, as it is natural for a study with ethnographic features, I observed the activities in the classroom as long as needed. During one semester, I was able to see the whole development from the introduction of robots to the smooth use of mathematical tools with the robots.

4.2.4 Contradictions and expansive transformations

As I have discussed in the theoretical framework section, contradictions can have the potential to change the activity development, which can lead to expansive transformations. The contradictions and expansive transformations are possible in collective multi-voiced activities, where subjects have their histories regarding tools. This kind of thinking is very suitable for collective activities with robots, where activity development is constituted by collective choices, and the activity development is not predictable beforehand. That can be challenging for the teacher, given that the activity development with robots is not predictable. And so, learning processes with robots cannot be studied with standard learning theories where students are seen to acquire stable skills and knowledge, which are identified by a teacher beforehand (Engeström, 2005). As mentioned earlier, learning is seen in this study as the development of a collective object instead of changes in individual knowledge or features.

Engeström (2005) created his methods for studies discussing expansive learning processes at work. The need for using this methodology usually stems from a situation in a workplace where someone is questioning the existing standards in practice. Due the questioning, there may arise contradictions in activity systems or between different activity systems. Contradictions create new activities and new models instead of supporting the old standards. Following its creation, the new model, or the new pattern, is examined and after that implemented before it can be taken as a new standard. During this process, different contradictions may arise on different levels, and different activity systems.

So, this kind of methodology is used in situations where there are problems with pre-existing standards for work practices. As the programming and robot-based activities are new in the mathematics classroom, there are no existing standards yet in use. Thus, the methodology created by Engeström (2005) cannot directly be utilized in this study. However, the five principles of CHAT are still useful for this study. Based on the reflections with the five principles in CHAT, I have developed a methodology where I concentrated on the different components in the activity system analysis.

I have now reflected the central aspects of focused ethnography and CHAT in light of my data collection and analysis. In the following subsection, I will present the processes among my data collection and analysis in more detail.

4.3 The data and methods

In this subsection, I will introduce my data collection process in detail by also introducing the most central parts of the data used in this study. The data-analysis process is also discussed in detail. Below is an introduction to the data sample used for articles 2 and 3.

4.3.1 The sample of this study

In Article 1, the sampling was conducted as a systematic search. This systematic search process with different phases is described in detail in Article 1. In this subsection, I will concentrate on the sampling for articles 2 and 3, both of which are based on the same data-material.

4.3.1.1 Sampling in articles 2 and 3

As I have mentioned in the introduction and earlier in this section, the empirical part of this study is conducted in one lower secondary school in Norway as part of an elective course called “technology in practice.” The teacher who introduced robots in his classroom was interested in programming integration, but did not have programming education. The focus in the data collection and analysis was on student groups of three students, aged 12 to 13. Several practical issues along with the design of this study influenced these solutions. In this subsection, I will discuss the justifications and drawbacks of these solutions.

Firstly, a Norwegian school suits well in this study because Norway is in the planning phase of programming integration. Thus, there is a need and interest in this theme, locally.

Secondly, the most central aspect of choosing a teacher, was that the teacher was interested in programming integration. In the beginning of this project, I had just moved to Norway, and I did not personally know any schools or mathematics teachers in this country. I did not know the teacher in beforehand; he was recruited for the study through my colleague, who knew him. I was looking for a mathematics teacher who wished to participate in this project and wanted to integrate programming and robots into their classroom. Thus, in the beginning, my only criteria were a mathematics teacher in a lower secondary school who was interested in integrating robots in their classroom. I also contacted another teacher through my colleague, but the teacher of this study was the first to answer affirmative when I called him. My colleague had already spoken to him beforehand and asked if he might be interested in participating in this programming project.

After selecting this teacher, I found out that he did not have a programming education, but that he was interested in knowing more about programming and robots, because programming is going to be integrated into mathematics curriculum in Norway. Thus, the teacher recruited for this study represents a teacher who is interested in programming and robots, but who did not have a programming education; because he was interested in digital technology in education, he was about to start teaching an elective subject called “Technology in Practice.”

Thirdly, the lower secondary school level was suitable because in the Norwegian context, the discussion about programming integration with mathematics mostly concerns lower secondary school.

Fourthly, the choice that this study is conducted as part of an elective study, instead of as a part of mathematics education, was a conscious one. I made this choice together with the teacher; he was skeptical about how the robots would work together with the mathematics curriculum, and he suggested the possibility of introducing the robots in his elective subject. He justified his proposal on the grounds that the robots were still very unfamiliar for him, and he did not know how to connect the robot-related activities with the mathematics curriculum, as programming was not yet part of the mathematics curriculum at that time in Norway. Still, he was open to finding out what happens in the classroom when robots are integrated therein.

He was also curious to know how programming was suitable with mathematics. Thus, we choose another subject from the subjects taught by him. The entirely open curriculum of an elective study provided a fruitful environment for innovative activity development without any pressure from the mathematics curriculum, or without time-pressure, as is often the case in mathematics classroom. Still, as the teacher was a mathematics teacher, he was able to combine different subject areas from mathematics curriculum in his elective subject classroom—technology in practice. Thus, I was able to investigate potential links between mathematics and programming activities, without needing to force the learning agenda.

As discussed in the Context chapter, students are more focused on digital technology than on mathematics, even if activities with digital technology take place in a mathematics classroom (Drijvers, 2018). Thus, it is to be assumed that it is also the case in other classrooms, as well. The elective classroom made it possible to view links between mathematics and programming activities from a neutral viewpoint. Thus, students did not have pre-assumptions to link their activities with mathematics, which made the potential findings of the links between mathematics and programming activities more valid. However, as the teacher in this study was a mathematics teacher, and he was also curious about the potential links between mathematics and programming activities, he contributed to linking programming activities with mathematics, which was interesting because one focus in this study was, indeed, on teachers' contributions to students' learning processes in mathematics. The teacher also knew that this project handled programming and mathematics, and as we first considered, together with him, the possibility of integrating robots in his mathematics classroom, he likely concentrated more on mathematics than he would have done in his ordinary technology classroom. Thus, he possibly acted as more of a mathematics teacher than a technology teacher during the data collection, even though the data was gathered in a technology classroom. Since the teacher was also a mathematics teacher, this study is relevant for mathematics education.

The drawback of the elective study approach is that it does not directly correspond to the situations in a mathematics classroom, where the curriculum and time-pressure are a reality. The student composition was also different than in a mathematics classroom. However, the aim of this study was to consider the potential links between mathematics and programming activities, and after that, to discuss the ability to transfer these activities to a mathematics classroom. During the data collection of this study, programming was not yet integrated in mathematics curriculum in Norway. Thus, there was a possibility that the programming activities were not transferable to mathematics classroom. However, as programming is going to be integrated in mathematics curriculum in Norway with the curriculum reform, the robot-based activities will be better suited in a mathematics classroom as such.

Finally, the decision to only observe one student group was based partly on practical reasons and partly on the research strategy. When I started my fieldwork, I wished to follow several groups working with robots with several video cameras. I started by videotaping several groups at the same time. However, I quickly realized that it was impossible to capture what the students were discussing when all the groups were working at the same time. Students were very active with the robots, and they moved quite a bit in the classroom, as well as outside of the classroom, in order to test and program their robots, which demanded a lot of attention from the observer and made it impossible to follow the discussions inside and outside of the classroom at the same time. Thus, it was impossible to follow several groups' activities at the same time. Thus, I ended up following three key student informants and the teacher. It was possible to get a detailed understanding of the students' collective learning processes by only observing one group of students and following them with the camera when they moved inside and outside of the classroom.

I made the student selection during the data collection. I first started by observing another group of students. This turned out to be impossible because the students in this group could not act naturally when video recording was on. They only fooled and made jokes without working with robots. Thus, I changed the sample group. The new group was able to act quite naturally despite the camera, and their actions became more natural in each session. In the following subsection, I will present in detail how I conducted the data collection with these key informants.

4.3.2 Data collection and my role as a researcher

Based on the reflection with the features in a focused ethnography, the five principles of the CHAT, literature review and earlier discussions about programming and robots in mathematics education, I conducted a data collection. The data of this study consists of notes from the three meetings with the teacher before the classroom observations, field notes and video recordings of classroom observations during one semester, a group interview with three students and a short questionnaire for all the students in the classroom. The primary focus was on observations. I observed activities in the classroom through video cameras to better understand everyday activities in the classroom when robots are integrated.

I started my data collection process by introducing Lego Mindstorms robots for the teacher. As mentioned in the introduction section, due to the visual programming environment, Lego Mindstorms robots are suitable at the beginning of programming introduction. The teacher was also interested in robots, even though he was not familiar with them; thus, the introduction of the robots happened on his terms. He wanted to take an active role already in the beginning. I only introduced the EV3-software for him, explained some basic programming figures in order to steer the robot, and provided some suggestions to find more information of robots on the internet. After my introduction, the teacher planned by himself and conducted an introduction of robots for 31 students aged 12 to 15. The teacher started by introducing the basic programming figures for the students in order to steer the robot motors. Other programming skills remained self-taught by students through the testing of the functions of different figures in the EV3-programming environment. The students worked in groups of 2-4 students. The task was often to drive a specific route with the robot. Students were also able to plan the route the robot was meant to drive by themselves. The assignments designed by the teacher were quite open; students got the opportunity to make their designs within the tasks, such as, what kind of track they would program the robot to drive. The open nature of the task enabled a free environment for activity development.

I observed activities in the classroom during one semester. As previously mentioned, I started by observing the whole classroom and the activities of different groups by using two different video cameras, one filming the whole class and one concentrating on the students' activities in more detail. After a couple of efforts, I started filming only one group of students with a single video camera. The choice to only follow one group of students allowed me to carefully follow classroom activities by taking into account all the components in detail.

Seven of the sessions were videotaped, five of them with the key informants. The duration of the data collection was determined during the data gathering period. It was the needed time to gain the needed information about the use of mathematical tools with robot-based activities.

During the videotaping, I concentrated on the students' activities in detail, as well as interactions between students and teacher. I aimed to capture all conversations and gestures in order to be able to analyze collective activities on a micro-level. The focus in my observations, video recordings and notes were on one group of three students, "Lucas," "Oscar" and "Jacob," and the teacher - my key informants. The recorded data material consisted of a total of over 12.5 hours of video recordings, which I stored securely.

At the end of the data collection, I conducted a group interview with a group of three students, three of my key informants. I showed them selected clips of the video recordings. I asked them open questions, and they could quite freely discuss their opinions of the tasks and choices during the activity development. In addition to this, at the end of the data collection, I asked all the students in the classroom to fill out a questionnaire. This questionnaire asked them, among other things, to (in their own words) explain what they have done with robots, whether they think they have used mathematics in the activities with robots, and what kind of mathematics in that case. However, this was only a supplementary material, the focus in the data collection and the analysis was on classroom observations. That was because the videotaped classroom observations provided detailed and valuable information about activities in the classroom and possible links between mathematics and programming activities. Instead of using other data sources, such as interviews or document analysis, I chose to go deep in my detailed observations on one group of students and teacher's activities on a micro-level by addressing communication, interactions, negotiations, gestures, and expressions, during the activity development in detail.

Four of the observed sessions were selected for analysis. The parts of these sessions, a total of 2.5 hours of videotaped material, were processed by a professional transcriber. These selected parts were interesting regarding the mathematical tools that were in use during robot-based activities. Less exciting parts such as the building of robots were left out from the transcription. The transcribed material did not include detailed descriptions about nonverbal communication and gestures. Thus, based on detailed video records, I complemented field notes with more detailed descriptions with different nonverbal actions, communication, and gestures, which gave me detailed comprehensive material to analyze. I observed the key informants' activities on a micro-level. I concentrated my observations on the activity development, and the effect of different components such as the role of the teacher, collaboration between students and different tools in use.

During the sessions, I concentrated on videotaping and observing. My role in the classroom was a moderate participant, which is between an active and a passive participant, based on the participation scale (Table 7) with five type of participation and involvement levels of researcher that was introduced by Spradley (1980). The moderate participant differs from the active and complete participant because the researcher does not attend all of the activities with the informants (Spradley, 1980). Furthermore, the role of a moderate participant is not totally passive, either. I had an active role in the beginning of the data collection, after which my role changed to being more passive, although not totally passive. Thus, as discussed in the introduction chapter, this study has features of an intervention study given that I had an active role in the beginning by participating in the introduction of the robots to the teacher. However, I did not participate in the teaching and guiding of students because I was interested to see how the activities developed between the students and the teacher. Thus, this study is only partly an intervention study. I sometimes discussed with the students at the beginning or the end of the sessions. The purpose was not to teach them; I only made some *small talk* in order to familiarize them with my presence in the classroom. My intention was not, in that phase, to study myself as an interventionist or act as an interviewer. My intention was instead to get my presence in the classroom as natural as it could be. As I mentioned earlier in this chapter, I started by following another group, who could not act naturally when the video recording was on. Thus, students' natural actions in front of the camera and my presence are not self-evident. That is also the reason why I wanted the students to get familiar with my presence. I also discussed with the teacher between sessions when needed in order to make my presence also natural for him. And so, even if I tried to make my presence seem as natural as possible, I cannot deny that my presence had some influence on the students and the teacher's activities. The influence of my presence is discussed later in this chapter in connection with the validity discussion.

Table 4. The participation types and involvement (Spradley, 1980, p. 58).

Degree of involvement	**Type of participation**
High	Complete
	Active
	Moderate
Low	Passive
(No involvement)	Non-participation

4.3.3 Data analysis

The data in this study is analyzed with abductive inference. The different modes of inference are often distinguished in social science in three different approaches: deduction, induction, and abduction (Danermark, Ekström, Jakobsen & Karlsson, 2002). While in deduction, conclusions are drawn from universal laws, and in induction, universally valid conclusions are drawn from a number of observations; the logic in abduction is "to interpret and recontextualize individual phenomena within a conceptual framework or a set of ideas. To be able to understand something in a new way by observing and interpreting this something in a new conceptual framework" (Danermark et al., 2002, p. 80). Thus, while deductive analysis is theory-based and inductive analysis is data-driven, abductive analysis is based on the circular movement between data and theory. Abduction provides an opportunity for deep understanding by connecting different ideas and knowledge with each other.

According to Danermark et al. (2002), the abductive analysis starts with a concrete empirical phenomenon. By relating that empirical phenomenon to the theory, we get a new interpretation thereof. G. Afdal (2010, p. 106) stated:

> Abduction is … a complex process of interpretation of material from a certain theoretical perspective and interpretation of theory from the understanding of data. The understanding of the material is dependent on the theory, but the material also contains implicit and multiple theories, which again change theory.

The abductive analysis provided an opportunity to gain deep knowledge about robot integration by detecting relationships and mechanisms between theory and observed activities in the classroom (Danermark et al., 2002). This is accomplished by "moving back and forth between data and theory" (G. Afdal, 2010, p. 106). I started the data analysis by watching all of the video-recorded sessions. I selected the sessions that were interesting as they related to the role of mathematics and the role of the teacher in students' activities with robots for further analysis. The selected sessions were the phenomenon that I wanted to interpret with activity system analysis in CHAT.

I watched the selected sessions again and wrote more detailed field notes with detailed descriptions about students' activities and actions in the classroom. Based on CHAT, I made a rough identification of different kinds of activity systems and more detailed, different kinds of phases in the development of these activity systems from the sessions. In this way, I roughly identified what the group of students were doing during these sessions. The different activity systems or phases in activity development in activity systems were, for instance "using the touch sensors to steer the robots," "designing the task," "designing the path for the robot to drive," or "determining the distance the robot has to drive." After this rough analysis, I went deeper by conducting a more detailed analysis wherein I coded the activities with detailed codes from the activity system triangle. I identified in detail the subject, objects, tools, rules, community, and division of labor in these selected activity systems. I also identified the ways in which these components evolved during activity development and how the components are related to and influence each other.

Due to this identification, I was able to discover how, for instance, the role of mathematics changed and transformed during activity development. First I figured out that mathematics did not have any role at all in the beginning; and after that, I identified that mathematics had a role as a tool; and after that, I came to understand that the role of mathematics changed to that of the object of the activity. In order to understand how different components influenced the change in the role of mathematics in students' activities, I focused on the relationships between different components; I also compared different sessions and activities with each other. Through different activity comparisons, I determined that when the teacher participated in the students' activities during the design phase and negotiated with the students, he was able to influence which mathematical tools were used. This kind of back-and-forth movements between data material and theory enabled me to gain a deeper and more detailed understanding of the links between mathematics and students' activities with robots. Thus, the circular movement between data and theory provided me with an empirical understanding of the potential links between mathematics and programming activities.

Furthermore, the theoretical analyses of empirical data provided new theoretical insights about the potential links between mathematics and programming activities. The abductive analysis provided a new theoretical insight, for instance, into how the role of the teacher influences students' learning processes in mathematics (see Article 3). I started the analysis by focusing on teacher-led or student-led approaches and figuring out which approaches were prevalent in different phases in the activity development process by discussing how the role of the teacher influenced the activity development. Based on the circular movement between data and theory, I found out that some of the sessions were more student-centric than others. I further found that the most fruitful activity development took place when the teacher and the students negotiated and collaborated towards a collective object. Thus, the fruitful approach was neither totally student-centric nor teacher-led, but something between these approaches.

As was observed, the abductive analysis combines both theoretical and data material-oriented approaches by jointly answering the questions: “What does the theory say about different events?” (Danermark et al., 2002, p. 95) and “What do the events say about theory?” (Danermark et al., 2002, p. 95). Thus, in the abductive analysis in this study, I approached the data, both data material-oriented and theory-oriented. I will discuss these processes in more detail in the following.

As I previously mentioned, I started the data analysis by watching all video-recorded sessions (sessions 2 to 8 were videotaped) and choosing the most exciting sessions regarding mathematical tools in use and the role of the teacher for further analysis. In this phase I chose the most exciting parts from my data based on my main goal in this study. In order to investigate the links between mathematics and programming activities I selected sessions, which were interesting regarding mathematical tools in use. Either mathematics was in use during the selected sessions, or the session was interesting because mathematics could have been in use but was not. The most exciting sessions for this study were sessions 4 to 7, where the teacher gave specific task assignments for the students to solve. During the first three sessions, students built robots and tested their essential functions. The teacher did not give any tasks to solve in this phase. Session 4 was the first session where the teacher gave students a task to solve. During sessions 4-7, students solved different kinds of tasks and needed different kinds of tools in order to do so. The role of the teacher varied during sessions 4 to 7, which was interesting for this study. Session 8, in turn, did not provide new information in addition to sessions 4 to 7, given that students continued with the same type of task needing the same type of tools than in Session 7. Sessions 4 to 7 are presented in Table 5 below.

Table 5. A short presentation of Sessions 4 to 7

	Session 4	**Session 5**	**Session 6**	**Session 7**
Task assignment	Program the robot to drive a particular route.	Program the robot to drive a specific route in competition with other groups.	Design a route and program the robot to drive it.	Design a route, drive it with the robot, catch a box, and drive back with the box.
A short presentation of activities	Students measured and calculated the distance the robot drove during one-wheel rotation.	Students wanted to use touch sensors in order to steer the robot. However, they had lacking programming skills. The task remained unsolved.	Students started by programming the robot to drive in a circle. The teacher suggested that students could drive a circle with a radius of 1 meter. One of the students was absent.	Students designed a new route and programmed the robot to drive it. Students had some difficulties with collaboration.
The use of mathematics	Students needed circle geometry and used a multiplication algorithm. However, they made an error with the multiplication algorithm.	The mathematical tools were not systemically in use during this session.	Students needed many mathematical tools in order to solve the problem. However, students made an error with the circle perimeter formula, which enabled fruitful learning sessions.	Students wanted to use the mathematics they learned in the last session for their new task at the beginning of the session.
The role of the teacher	The teacher advised students to calculate the tire circumference instead of measuring it.	The teacher was not present when students designed the use of sensors. The teacher was not able to help students because of lacking programming skills.	The teacher attended students' activities already in the beginning when students were designing their tasks by making suggestions. The teacher was involved in several phases during the session.	The teacher did not attend students' activities in the beginning of the session.

In order to get a more detailed understanding of my data material, and in order to make a detailed analysis of interactions and negotiations between students and the teacher, as well as gestures and expressions of students and the teacher, I re-watched these sessions and wrote more detailed field notes. I paid extra attention to the details in the students' activities, such as interactions, expressions, and gestures. The more detailed written data material with ethnographical features was more comfortable to handle in my analysis when coding the data material.

After that, I conducted a more detailed data analysis with the help of the activity system analysis. The data material was coded with codes based on CHAT. These were components of subjects, object, tools, rules, community, division of labor and outcome in activity system analysis. This was done in order to receive an understanding of the activity development.

The object of the activity was determined by identifying the collective aim of the subjects of the activity. I identified, for instance, what students were aiming to gain with the activity or what the students' drive in the activity was, such as to get the robot to act as desired. The object was something that each subject focused on and aimed towards collectively; the subjects of the activity shared the same object. However, as discussed in the Theory chapter, the object is a dynamical, multilayered, and complex component, and tensions may arise in the collective activities, and the subjects of the activity may have different objects during the activity development. The situation may only be temporary, or bigger changes may escalate in the activity.

Since the object of the activity was complex and multilayered, it was sometimes difficult to interpret the objects of the activities during the activity development, and whether all subjects indeed shared the same object. Thus, I did not find a seamless way to analyze what each student was aiming for at any given moment based on the video data. I aimed to analyze slightly larger lines related to the object development. I concentrated on negotiations between the students and between the students and the teacher, and the ways in which these negotiations influenced the object development. This enabled me to gain an understanding as to how the object of the activity was developing, who was included, and who was sharing the same object. According to my findings, the students and the teacher negotiated about the task design, mathematics, how to program the robot to drive a certain path, or how to solve a certain task. Thus, the different identified objects were: task design, program the robot to drive a path, solving the task, and mathematics.

Respectively, the subjects of the activities were determined by identifying who aimed towards the same object and hence were the students as a group and the teacher. The subjects of the activities aimed toward the activity with different kinds of individual actions. For instance, when the students aimed to program the robot to drive a path, they needed to conduct some calculations or measurements, and they also needed to program the robot. Consequently, they divided the problem into smaller parts, with one student conducting the calculations, one doing the measurement, and one programming the robot. The collective activity consisted of these individual actions, all of which aimed toward the collective object, that being to drive a path with the robot.

The different tools, which were in use, were identified with the help of the object of the activity. I determined what kind of tools the students and teacher used in order to research the object. The tools were separated from the objects of the activity by identifying the focus of the subjects. According to Engeström (2005) the focus can only temporarily be on tools. For instance, when the object of the activity was to drive a circle with the radius of one meter with the robot, the students needed mathematical tools in order to determine the distance the robot had to drive. The focus was temporarily on mathematics when the students conducted the needed calculations. They used, for instance, the circle perimeter formula in order to determine how long the robots had to drive, as well as proportions in order to uncover how much the robot had to turn. After the students obtained the needed results from their calculations, they used them in their programming in order to reach their object; hence, the focus was no longer on the mathematical tools.

The mathematical tools that were identified include different algorithms, formulas, circle geometry, and proportions. Also, different kinds of digital tools, such as programming tools, were identified, as well as different technical tools, such as rulers, calculators and whiteboards. Finally, the different skills of the students and teacher, such as programming skills, were also identified tools.

The different identified rules were the rules from the mathematics classroom and the free rules in an elective subject classroom environment. As such, the task assignments were rules in the classroom. Time limitations framed the activities in the classroom.

The community in this study were all students and the teacher in the same classroom. Students in the classroom were from different grade levels; the atmosphere in the community was quite loose. The component of division of labor included the different role of the teacher and students and collaboration between them. In Figure 13, I have summarized the different codes used from my data.

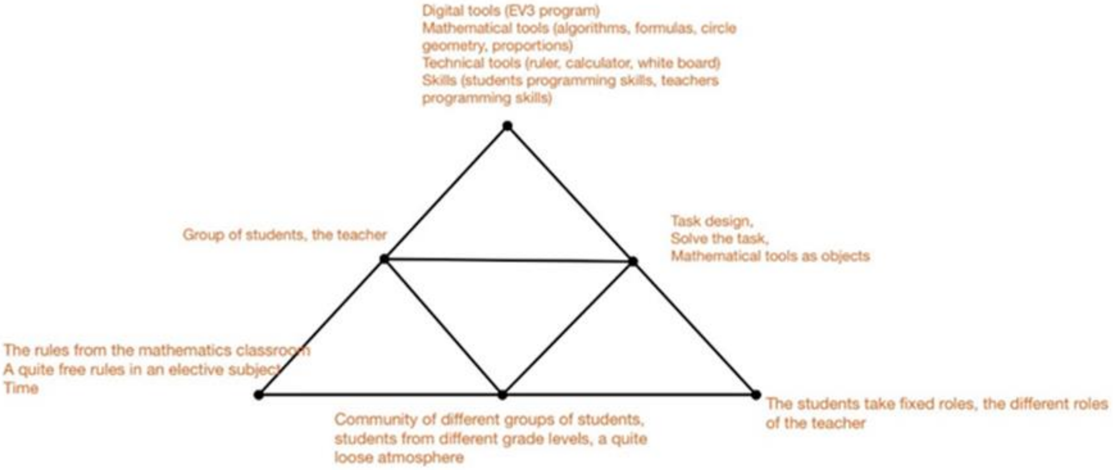

Figure 13. Summary of the codes.

After watching the videos several times, writing field notes and coding, I had quite a comprehensive understanding of the data of this study. I continued by drafting the development between different activity system models in order to find connections between different activities, codes, and sessions. This was done to get a comprehensive understanding of activity development and the links between mathematics and other components in the activity systems. The particular focus was on the use of different tools, the object development and the role of the teacher through division of labor during the different phases in the activity development.

In the next phase of my analysis, I concentrated separately on the use of mathematics in order to answer the research question in Article 2. Respectively, I concentrated on the analysis of the role of the teacher in order to answer the research question in Article 3. The most interesting sessions for Article 2 were sessions 6 and 7. Respectively, the most interesting sessions for Article 3 were sessions 5 and 6. The analysis of articles 2 and 3 are discussed separately and in more detail below.

4.3.3.1 Analysis in Article 2

The focus in Article 2 was on the mathematical tools in use. The analysis in the article is based on sessions 6 and 7 because mathematical tools were in fruitful use during these sessions. The activity development was also fruitful for the learning of mathematics. I will shortly present how we started to analyze the activity developed during sessions 6 and 7.

The use of mathematical tools is connected with the other components in activity system analysis, especially with the object of the activity. Thus, we concentrated the analysis on object development during different sessions and how object development affected the use of mathematical tools. According to our analysis, the activities developed during sessions 6 and 7, from task design to expansion of the object. During session 6, the students' object was first to program the robot to drive a circle with a radius of one meter. The students needed mathematical tools in order to reach their object. However, the students made an error with a mathematical tool, which influenced the activity development. Because of the activity development, mathematics became an object of the activity, i.e. mathematics became a drive and direction in students' activity.

The expansion of the object is operationalized by identifying expansive development in the object of the activity. The expansion can arise when the previous object is applied in a new, broader situation, and the previous object is part of a new, broader object. During the activity development, students became excited about their learning in mathematics. At the beginning of Session 7, the students were willing to apply their learning in a new, broader situation and the object of the activity expanded. In this study the students wanted the robot to drive along a circle of a different size than last session as a part of their new task. They were excited about Session 6, where they learned how to program the robot to drive in a circle with a certain radius. Thus, they wanted to apply their learning in a new, broader situation and the object of their activity expanded. According to Engeström (2005) the expansive transformation gives new, broader possibilities than the previous activity.

We have divided the activity development during sessions 6 and 7 into four different phases. Table 6 is a summary of how the use of mathematical tools and object of the activity developed sessions 6 and 7 from task design to the expansion of the object. During the activity development, mathematical tools were in use, mathematics became the object of the activity, and finally the object of the activity expanded. However, the activity development was constituted by different components, such as mathematization of the object and the error that the students made with mathematical tools. The analysis of the activity development is discussed in more detail in Article 2.

Table 6. The activity development during sessions 6 and 7 (retrieved from Article 2)

	1. The task design	**2. The use of mathematical tools**	**3. Mathematical tool as an object**	**4. Expansion of the object**
The object of the activity	Students started by programming the robot to drive a circle. The teacher mathematized students' object by negotiating with students.	The mathematized object, to drive a circle with the radius of 1m, enabled the use of mathematical tools.	Because of the error students made with the mathematical tool, mathematics became the object of the activity.	Students wanted to use their learning from the last sessions in their new task design. The object of the activity expanded.
Mathematical tools in use		Students used different kinds of mathematical tools in order to reach the object. However, they made an error with the circle perimeter formula.		Mathematical tools were in use again because of the new mathematized object.

4.3.3.2 Analysis in Article 3

Article 3 discusses how the role of the teacher influences the students’ learning processes in robot-based activities involving mathematics. The analysis in Article 3 is based on the comparison of the role of the teacher during sessions 5 and 6 because the role of the teacher varied remarkably during these sessions. I started the analysis by comparing different components in the activity systems during these sessions. I identified differences in different activity system models in sessions 5 and 6 by comparing the object development during these sessions. My comparison was based on codes, which I presented above. I found that during Session 5 the teacher was not present at the beginning of the session when the students negotiated their object. During Session 6, the teacher negotiated the object together with the students.

I continued by comparing different tools in use during sessions 5 and 6 based on the codes developed. I uncovered that, during Session 5, mathematical tools were not in use, while during Session 6, mathematical tools were systematically used. Regarding the programming tools, students had problems with these during Session 5 but not during session 6.

Furthermore, I compared the codes regarding the division of labor during the sessions. I found out that students had difficulties with collaboration during both sessions. During Session 5 the problems with collaboration were unsolved, while during Session 6 the problems with collaboration were solved with the role of the teacher.

The comparison between sessions 5 and 6 is visible in Table 7. After I had identified the differences between sessions 5 and 6, I analyzed how the role of the teacher influenced these differences. This was done by analyzing relationships between these components in the activity system model with the role of the teacher. I conducted a more detailed analysis of how the role of the teacher influenced the object of the activity, the mathematical tools in use, and collaboration between students during these two sessions. According to my findings, the teacher was able to influence the activity development through negotiating with the students during the activity design phase. During the session where the teacher negotiated the object together with the students (Session 6), mathematical tools were in use and the collaboration between students worked. More detailed analysis and the results are presented in Article 3.

Table 7. The comparison of sessions 5 and 6 regarding the role of the teacher (retrieved from Article 3)

	Session 5	Session 6
Object	The teacher was not present when the students negotiated the object	The teacher negotiated the object together with the students
Mathematical tools	Students did not use mathematical tools	Mathematical tools were in use
Programming tools	Students had problems with programming. They could not solve these problems	Students did not have any problems with programming
Collaboration	The students had difficulties with collaboration	The students had difficulties with collaboration; with the help of the teacher, the difficulties resolved
Outcome	Students did not complete the task	Students completed the task

4.4 The quality of my study

I have now presented and reflected my data collection process and analysis methods in detail. As discussed above, the components in the study design are strongly connected with each other. In this section, I conclude, the link between the study design, study strategy, and CHAT with Maxwell's (2005) Interactive Model of Research Design (see Figure 11). As discussed at the beginning of this methodology section, Figure 6 shows how different components in the study design depend on each other according to Maxwell's (2005) model. The effect of CHAT is visible in the layout of the research questions, choice of the methods, and conceptual framework. A focused ethnography as a research strategy had a strong influence on the data collection process in this study. In this section, I will reflect how these components affected the quality of the study. In the following subsection, I will discuss the validity, reliability, generalization and ethical questions of this study.

4.4.1 Validity, reliability and generalization of this study

As Maxwell's (2005) Interactive Model of Research Design (see Figure 11) shows, the validity of a study depends on other components in the study design. The validity of the study is also dependent on how different components in the study design fit together. Thus, the validity of qualitative studies is a complex component with possible threats at varying levels and layers. Earlier in this thesis, I had discussed that other components in the study design model fit together and are in relation to each other. In this subsection, I will discuss some of the points regarding how the methodological choices, both in data collection and data analysis, affect the validity of this study.

The data analysis of this study is based on an abductive approach, which has been criticized as it does not provide overall knowledge (Danermark et al., 2002). However, the aim of this study was not to find the overall knowledge regarding robot integration in mathematics learning; the aim was to gain deeper understanding at a micro-level, by discussing links between mathematics and programming activities, in which abductive analysis is suitable.

Another limitation of the abductive analysis is that there are no fixed tools to discuss the validity of the study. And so, Maxwell (2005) points to two critical threats in data collection and data analysis processes to consider when reflecting the validity of qualitative studies. The threat regarding the data collection process concerns *reactivity*, which refers to the need to understand the influence of the presence of the researcher. The influence of the researcher is impossible to eliminate; thus, Maxwell (2005) points out the importance of understanding and use it. As previously discussed, my role during the data gathering portion of this study was between passive participant and total participant observer. Students and the teacher were very aware of my presence, which influenced their activities.

I influenced the teacher in the beginning by introducing robots for him; because the teacher was a novice with programming and robots, the robot introduction sessions before the classroom sessions certainly had an influence on him. I concretely introduced Lego Mindstorm robots and the EV3-software that was needed to program the robot to him. I also briefly introduced the basic programming figures needed to get the robot to drive, and I gave him tips on different internet sites, such as the Lego education sites, from which he could get ideas for teaching. Certainly, this project and the theme of this project, my role as a researcher, and my background as a mathematics teacher had an influence on how the teacher approached the project and attended to the role of mathematics during this project.

I did not, however, push him with advice on teaching. The teacher was motivated to test the robots by himself, and I left the study of other programming skills up to him. The teacher was very curious to test the robots, first by himself, and soon also with his students; he did not ask me questions, and I did not want to push my advice. I was willing to help him more, and I also offered my help to him, but he made his own decisions about how and when to integrate robots in his classroom. The teacher wanted to make his own plans in his classroom, and he seemed to be comfortable and self-confident, even though he was a novice in programming. I tried to be as open as possible and to only act according to the needs of the teacher. The number of required meetings was mainly determined by the needs of the teacher, but he seemed to be ready quite early to test the robots with his students. The free environment in the elective study made it possible for the teacher to make free choices regarding how he would introduce the robots to the students. The teacher also seemed quite confident and enthusiastic to introduce something new to his students. I also think, because of his busy everyday teacher life, he had to make quick decisions. Even though I briefly introduced the robots for the teacher, our meetings mostly focused on practical matters, such as when and how the data collection could take place.

In conclusion, even though I as a researcher had a power relationship with the teacher based on my interests, my background, and this project, I tried to influence him as little as possible as to his decisions to integrate robots into his classroom. I did not want to push it, and the teacher made his own decisions regarding the integration of robots in his classroom. Furthermore, even if the short robot and programming integration that I conducted influenced the teacher, I did not teach the teacher anything he could not have learned from a colleague, by finding out for himself, or by participating in an event for teachers where robots were briefly introduced.

It is possible that my presence and that of the video camera made students act differently. Students most likely tried a bit more or joked a bit more than usually. On the other hand, even if the situation in the classroom was slightly unrealistic due to my presence, the atmosphere in the classroom was loose and free. The students joked and laughed, and they also dared to refuse to do the assigned tasks if they were not in the mood. They also discussed other things than the technology or robot classroom; they discussed, for instance, their test results from other subjects. I tried to make my presence as pleasant as possible by being familiar with the students by asking before and after the lessons how they are doing and whether they enjoyed working with the robots. I did not interrupt the students while they were working during the lesson; I simply followed them with my video camera when they moved inside or outside of the classroom in order to test the robot in a wider space. This, of course, affected student behavior. However, speculating about this is awkward. Nonetheless, the core activities were genuine even if they were activities with the presence of myself and the video cameras. To rectify this threat, as I have mentioned, I chose a specific group (sample) because they did not mind the presence of the video camera and myself as much as the other group of students did.

Furthermore, as I discussed with students in the beginning (or at the end) of the sessions in order to get them more familiar with my presence in the classroom. Thus, I argue that my presence became more natural for the students and the teacher during the semester. The most interesting learning sessions regarding the research questions were the last sessions where the focus was not so much on my presence. During these sessions the students did not pay too much attention to the camera. It is visible in the data that the students did not look at the camera. Instead, they spoke and looked at each other, and they concentrated on the robot, the computer and the whiteboard where they were writing their calculations. However as Lomax and Casey (1998) argued, the influence of camera cannot be neglected. The researcher with a camera always has a certain influence on the data. Still, as I mentioned, the goal was to look for potential links between mathematics and programming activities, and I was able to find real connections despite the presence of the camera.

Another threat, according Maxwell (2005) concerns the researcher’s *bias*, which refers to the possibility that research results can distort due the researcher's preconceptions, values and theories. Furthermore, according to Danermark et al. (2002) the abductive analysis process is dependent on the creativity and imagination of the researcher. Thus, the researcher's qualities are relevant to the validity of the research. According to Maxwell (2005) the idea is again to understand the influence of these components for the results of the study.

As I have discussed in the introduction, I had some preconceptions about programming integration through my background as a mathematics teacher. However, my preconceptions about the role of mathematics in programming integration were not that positive. Thus, I did not have big expectations about the potential links between mathematics and programming activities. And so, I have controlled my preconceptions through my role as a researcher, through the duration of the data collection and through the theory. Given that I participated mostly in the beginning when I introduced robots for the teacher and after that I did not participate in teaching activities except by taking video recordings and observing the activities in the classroom. The duration of the data gathering was long enough to reduce my preconceptions, which were rooted in a Finnish school context. Furthermore, as I have discussed in the methodology chapter, the theoretical framework of this study is strongly connected to the other components in the study design. With strong theory use, I was able to use professional analytical categories instead of my own, which minimizes the influence of my preconceptions.

On the other hand, my background can be seen as a positive factor for the validity of this study. First, my background as a mathematics teacher was the foundation from which I became interested in this theme firstly, and it also enabled me to set realistic goals for this study, because the theme of the study was familiar to me. As I mentioned earlier in the text, as a mathematics teacher, I was skeptical about the links between mathematics and programming activities. This enabled me to approach the data and the results of this study in a critical manner. I did not have any preconceptions that programming activities can be easily linked with curriculum mathematics. Second, due to my background as a teacher, classrooms as a research environment was already familiar to me. This helped me to understand and interpret complex classroom activities in a creative way. Third, my Finnish background helped me to distance myself from the Norwegian school reality, and vice versa. Even though I carried my teacher background with me, the Norwegian school allowed me to distance myself from that role and to take on the role of a researcher, which enabled me to be more open-minded about what could happen in the classroom.

According to Maxwell (2005), generalization and reliability are challenging parts during a validity check in qualitative studies. There is a risk that qualitative studies focus too much on one special case. That is also a potential risk in this study because this study concentrates only one group of students and one teacher's activities. But, the research background in this study corresponds to an ordinary situation in an ordinary school in Norway; no additional arrangements were made, expect my presence in the classroom, and a video camera.

The sampling of this study was small. I aimed to follow activities in the classroom on a micro-level. The micro-level analysis would not have been possible on a broader scale with one researcher in the classroom. Also, a micro-level analysis made it possible for me to describe the learning processes regarding the use of mathematics in detail, which contribute to detailed conclusions about the role of mathematics and the role of the teacher in robot-based activities in the classroom.

Finally, according to Maxwell (2005), essential in a successful qualitative study is cohesion between the components of the design of the study. In this methodology section and the other sections in this thesis, I have justified how different components in my study design fit together and are in relationship with one another. And so, in addition to the different components in the study design being cohesive, each component should include ethical concerns. In the following subsection, I will discuss some ethical reflections drawn from this study.

4.4.2 Ethical reflections

According to Tangen (2014), ethical considerations in a study should include reflections about the protection and benefit of participants, on the one hand, and the internal and external quality of the research on the other. Tangen (2014) divided the ethical reflections into four main categories. First, she argued that ethical reflections in a study begin in the study design phase when discussing the justifications of the study and the relevance for the field and practice. I have discussed and justified the design of this study in both the Introduction and Methodology chapters. The aim, focus and relevance of this study is justified with earlier studies and with my own interests in the Introduction. The focus on students' learning processes in classroom and also the focus on the role of the teacher during these processes are justified. In the Methodology section, I have discussed the data-gathering methods used in this study in order to capture the learning processes in the classroom. Based on reflections between the different data gathered in this study, the videotaped classroom material with the key informants was chosen to be the main material in the data analysis; this was because the video material provided detailed information on a micro-level on the activities with different kind of individual actions, collaboration, interactions between participants and tools, negotiations, and gestures in the classroom.

Tangen (2014) emphasized that ethical considerations should be present during the recruiting processes of participants by reflecting on what information should be given to the informants and to the research community when discussing the aim of the study and the research problem therein. As programming is going to be integrated in the Norwegian mathematics curriculum, it was natural for me to highlight to the teacher that the focus of this study is mathematics and programming; as he was a mathematics teacher, he was also curious about the potential links between mathematics and programming activities. Mathematics was a natural starting point in our conversations, and we both understood the complexity of linking mathematics with other activities. The teacher told me directly that he does not know programming and robots well enough that he could directly integrate them into his mathematics classroom, but he was curious about the links between programming and mathematics and was willing to test robots in his elective class.

Furthermore, when I introduced myself to the students, I also introduced the project name, which was "Programming in Mathematics Education." The project name and the focus of this study were also stated in the letters that were sent to the students' homes (see Appendix 1). I also told the informants that my field is mathematics education, and that I have worked as a mathematics teacher.

There is a possibility that my openness influenced the students' and teacher's focus in the classroom. The teacher probably wanted to have mathematical tools as a part of the classroom activities in order to "succeed" in this study. The students may have also had mathematics on their minds during the sessions because they knew what the project involved. However, the discussions and debate about integration of programming in mathematics was ongoing in Norway in the beginning of this study, and the teacher was aware of this debate, and he was curious about the potential links between mathematics and programming. I think this set-up seemed natural for the students, because their teacher was a mathematics teacher, and they were used to discussing mathematics with him. If this study had been part of a mathematics classroom, the consideration between mathematics and programming would have been even stronger. Thus, I can conclude that the influence of the openness about the study's focus was not greater than it would have been if the research had been carried out as part of the teaching of mathematics.

Second, Tangen (2014) discussed the ethical reflections in data collection and analysis. There are several ethical issues to consider in the data collection and analysis in this study. As this study concerns classroom activities with under-aged persons and sensitive, videotaped material are gathered, particular ethical confirmations during my research process were necessary. Firstly, I asked for written permission from the custodians of the students for video recordings. The letter I sent to the custodians is in Appendix 1. Secondly, I asked permission for the gathering of sensitive data from the Norwegian Centre for Research Data, from where I received permission and complete orders regarding how to handle and store sensitive material (Appendix 2). These rules were followed strictly, for instance, in matters related to the storage of data relating to students.

When it comes to the data analysis, I have reflected on the tools and methods used in the analysis. The use of the abductive analysis method enabled both the data-material-oriented and theory-oriented approaches in data analysis with circular back-and-forth movements between the data and the theory. The use of CHAT as an analytical tool in this study is justified in the Introduction section and in the beginning of the Theoretical Framework section. The justifications about how CHAT is used as an analytical tool is reflected and discussed in the Methodology section.

Third, Tangen (2014) argued that ethical considerations are essential when reporting and publishing a project. When thinking about the protection and benefit of the participants, the anonymity of the informants is crucial. The school, the teacher and students are anonymous in this study. The names of students were changed, and I have not disclosed the school's name or location. This was also taken into account in the published articles, where the actual student or teacher names are not given. The articles only name the Norwegian school context, the gender of the teacher and the students, and provide some details about the teacher's background, such as the fact that he did not have programming education. This is assumed to be a fairly common situation in Norway, where programming is not yet part of the curriculum. Thus, this information did not personalize the teacher.

When thinking the internal and external quality of the study in publications, Tangen (2014) asserted that it is important to consider how the results can be creative, precise, and systematic and how the results are discussed with relevant research. Regarding my data analysis, I have consciously minimized concern of my conclusions by discussing regularly and sincerely about my data and my ideas with my supervisors and colleagues. I have also presented my ideas in some seminars and conferences. After my ideas and writings, I have often gone back to my original data and re-watched the videotaped material to secure my ideas and thoughts.

Fourth, Tangen (2014) argued that the role of the researcher brings ethical challenges into the study. I have discussed the issues regarding this point earlier in this section by addressing the importance of being aware of possible influences. I have discussed the influence of my role as a moderate participant in the data collection for this study, which was mostly visible in the beginning of the data collection, when I participated by introducing Lego Mindstorm robots for the teacher. The awareness of this influence is discussed earlier in this section.

In summary, as this methodology section describes, my study process has been multilayered with intertwined components. As a result, from my data gathering and analysis, I have written three separate articles, which I present in the following section.

5. Summary of the articles

In this section, I will present a summary of articles 2 and 3 as Article 1 is summarized in the introduction section. All three articles jointly contribute to the overall research question, each with its' own perspective and contributions. Article 1 was a literature review contributing by highlighting the potential benefits of programming in mathematics education and unsolved questions regarding them. Based on the literature review, Article 2 addresses the use of mathematical tools in robot-based activities and Article 3 the role of the teacher in students' learning processes in mathematics upon the integration of robots.

5.1 Article 2: Learning mathematics through activities with robots

This article is an empirical contribution to the discussion about links between robot-based activities and the mathematics curriculum. The article aims to take a closer look at the use of mathematical tools in robot-based activities by answering the question:

What is the relationship between mathematical tools and objects in robot-based collective student learning activities in secondary education?

This article describes the activity development during data gathering sessions 6 and 7, where the focus was on one group of three students' activities with robots. Activity development is divided into four different phases (see Table 6). The first phase was the design phase where the teacher suggested to the students that they could program the robot to drive in a circle with a radius of one meter. Before the teacher's suggestion, the students were programming the robot to drive in a circle without specifying its size. They used the trial and error strategy in order to solve the problem. The mathematized object, to drive a circle with the radius of one meter, required mathematical tools to be used. The mathematical tools were in use in the second phase. Students used, for instance, the circle perimeter formula and proportion in order to find out how long the robot had to drive and how much it had to turn. However, students committed an error with the given circle circumference formula, using the radius rather than the diameter. Thus, the robot drove only half a circle. After the teacher's steadfast negotiations with the students, the students started to find out why they needed to double their answer, mathematics became the object of the activity, which was the third phase.

In the last (fourth) phase, the object expanded, because the students were willing to use their collective learning of mathematics in a new, wider situation. The students wanted to apply their learning from the last session, and they wanted to have a circle track as part of the new route for the robot to drive in their new task.

The task assignments during these sessions were quite student-centric, and the students were able to design their own paths for the robot to drive. However, the activity development was not totally student-centric; the teacher participated in several different phases, mostly on the activity design phase, where he negotiated the object together with the students. Furthermore, with his steadfast negotiations the teacher influenced on the object development, when mathematics became the object of the activity. Thus, the activities were not totally student-centric or entirely teacher-led; they were something in-between. The approach was based on collaboration between the teacher and the students.

As a conclusion of this activity development, students used and learned mathematics collectively during their robot-based activities. However, the use of mathematics and learning was constituted by these turning points, which could not have been predicted. However, the activity development described in the article proves that the use and learning of formal mathematics are possible through robot-based activities. We argue that activities described in the article provide possibilities for curriculum mathematics to be the object of students' classroom activities.

5.2 Article 3: Role of teachers in students' mathematics learning processes upon the integration of robots

This article is an empirical contribution to the discussion about the role of the teacher in integration of digital technology when the teacher does not have an extensive training in programming. The aim of this article is to answer the question:

How does the role of the teacher in robot-based activities influence students' learning processes in mathematics?

This article compares the role of the teacher during two different data collection sessions (sessions 5 and 6), where the role of the teacher differed a lot. The detailed analysis concentrated on the relationships; the role of the teacher and the use of tools; the role of the teacher and object development; and the role of the teacher and collaboration between students was conducted. Table 8 summarizes my findings regarding the influence of the role of the teacher and object, tools and collaboration. According to my findings, the collaboration between students and the teacher enabled fruitful activity development with fruitful object development, mathematical tools in use, and successful collaboration between students. During Session 5 (Session 1 in the article), the teacher was absent when students negotiated the object, but during Session 6 (Session 2 in the article), the teacher negotiated the object collectively with the students.

Table 8. Summary of the findings regarding the relationships between the role of the teacher and object development, the tools in use, and collaboration between students (retrieved from Article 3)

	The Role of the Teacher- Object	**The Role of the Teacher- Tools**	**The Role of the Teacher- Collaboration**
Session5 **(Session 1 in the article)**	The teacher was not present in the design phase of this session when students negotiated their object. This influenced activity development, such as tools in use and collaboration among students.	Students did not have the tools required to obtain their object. This could have been avoided if the teacher was involved earlier in object negotiation. The teacher did not have the tools required to guide the students in this phase.	At the end of the session, students' collaboration was not successful. This was a consequence of activity development that was based on the object of the activity and tools in use. This could have been avoided had the teacher been involved in the previous phases with some of his questions as a guide.
Session6 **(Session 2 in the article)**	By making suggestions based on the students' original activity, the teacher and the students negotiated an ordinary object together. This had an influence on the development of activity, such as tools in use and collaboration among the students.	Students had the programming tools required to obtain their object. Also, mathematical tools were in use. The tools applied were based on the object negotiated with the teacher. Moreover, the teacher had the tools needed to guide the students, such as his mathematical and pedagogical knowledge.	Students had difficulties with collaborating among themselves in the beginning. This situation was solved by the teacher's guiding questions and negotiation at the beginning of the session. After the collective object was negotiated, collaboration among the students was successful during the development of the whole activity. This was based on the object of the activity and tools in use.

In summary, the findings show that the activity developed fruitfully when the teacher negotiated a collective object with the students in the activity design phase. This differs from a traditional classroom model, where a teacher often gives fixed tasks for the students to solve (Engeström, 2008; Martinovic, Freiman, & Karadag, 2013). Furthermore, it differs from a student-centered approach, where students can design and create their objects. A total student-centered approach does not work if the teacher has lacking technological knowledge. In such approach, students are on their own if the teacher is not able to help them. That was visible in this study. In the successful robot integration in this study, both the teacher and the students used their knowledge properly. The teacher used their pedagogical knowledge, which compensated his lack of programming knowledge when he was negotiating with students. At the same time, students contributed with their knowledge and ideas in a collective object negotiation.

6. Discussion

This study has addressed programming and robot integration in mathematics education. In this chapter, I will answer the overall research question:

What are the links between mathematics and programming in classroom activities?

We have addressed this question firstly by writing a literature review article with the research question: *What is the educational potential of programming in mathematics education?* Based on the review and earlier studies, this study aimed to answer two more detailed research questions. Firstly, in Article 2:

"What is the relationship between mathematical tools and object in robot-based collective student learning activities in secondary education?"

Secondly, in Article 3:

"How does the role of the teacher in robot-based activities influence students' learning processes in mathematics?"

In this section, I will discuss my findings across the three articles to justify the main claim in this study namely that the links between mathematics and programming activities have a transformative potential in mathematics education through an active and negotiating teacher role. After that, I will reflect on how the main findings of this study contribute to the debate considering transformational potential of integration of digital technology in mathematics education. Finally, I will discuss possible implications of the findings of this study for policy and practice.

6.1. Findings across the articles

According to our literature review, earlier studies discussing programming and mathematics education address students' motivation to learn mathematics and improvement in students' mathematics learning. From this review, it was determined that programming integration has the potential to motivate students to learn mathematics and improve students' learning in mathematics at least in some of the cases. Programming activities can provide outside of the classroom connections in mathematics education (Ke, 2014; Leonard et al., 2016). However, the results cannot be generalized. Furthermore, we found that further discussion is needed regarding how the different components, such as the role of the teacher and collaboration between students, in students' learning processes influence students' learning. The role of the teacher is interesting, because ordinary mathematics teachers have the responsibility of programming integration. The teacher's role in itself is not very interesting, but the teacher's relationship with students, tools, objects, division of labor and collaboration gives information about the whole learning process in the classroom. Thus, for more in-depth understanding of links between mathematics and programming activities, the entire collective learning process is interesting, as opposed to merely measuring students' motivation and learning at the end of the process.

To build on our findings from the first article, a more detailed understanding of the links between mathematics and programming activities was sought by concentrating on students' collective learning processes with robots on a micro-level. In this study, I viewed learning as a collaborative process instead of an individual result. I concentrated on interactions between students, the teacher and programming tools and robots. The focus was on different relationships such as relationships between the role of the teacher, tools, and objects of the activity; teaching was seen as a relational activity.

In Article 2, we discussed the relationship between mathematical tools and object in robot-based collective student learning activities in secondary education as the use of mathematics was constituted by the object of the activity. A mathematized object enabled the use of mathematical tools. Furthermore, the error that the students made with a mathematical tool enabled object development and mathematics thus became the object of the activity. After certain turning points, the students were willing and capable to apply their learning in a new and broader situation, the object of the activity expanded.

The analysis in Article 2 indicates that robot-based activities both hamper and enable the use of formal mathematics. The open nature of the activities made it challenging for students to remember formalities regarding mathematical tools used during the activities. On the other hand, after mathematics became the object of the activity, the students paid attention to formal mathematics together with the teacher. Thus, the students had an opportunity, and they were motivated to learn formal curriculum mathematics.

Article 3 addresses partly the same learning processes by discussing the role of the teacher in these processes. The article compares the teacher's role in two different sessions where the role of the teacher was different. It was argued that the fruitful integration of programming and robots took place when the students and the teacher worked collectively towards the same object. The activity developed in a fruitful way when the teacher and the students collaborated by negotiating the object of the activity together. Through object negotiations, the teacher was able to mathematize the students' objects and got them to use systematically mathematical tools in their robot-based activities.

I will summarize the findings across the articles with Figure 14, found below, which is an application of the activity-system model in the successful robot integration in this study. This activity-system model will be discussed by introducing the bottom of the triangle and then moving upwards by discussing the subject and the object of the activity. Finally, I discuss the use of tools. The classroom rules (1) were entirely free in this study: students had an opportunity to design their tasks. However according to my analysis, an approach that was student-centric was challenging in the situation, where the mathematics teacher did not have programming education (Article 3). In a student-centered approach, the teacher faced difficulties with helping students with their problems regarding programming. I argue that the collaboration between students and the teacher enabled successful activity development (2). The collective object negotiation (3) between students and the teacher was the initiator for the successful activity development in this study (Article 3). The teacher mathematized the object by negotiating with the students, which contributed to the systematical use of mathematical tools (4), which followed a fruitful activity development where the role of mathematics changed as well, from tool to the object of the activity.

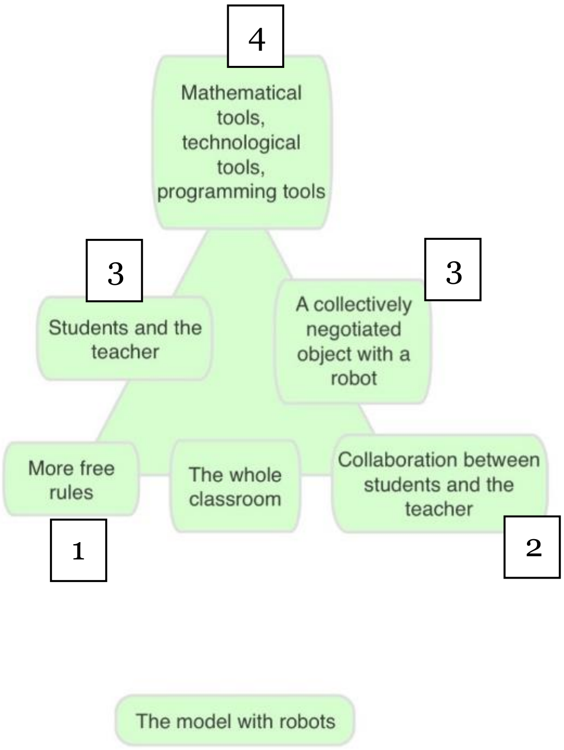

Figure 14. The model of successful robot integration from the perspective of the activity system analysis in CHAT (Engeström, 1987)

According to my findings, mathematics can be linked with programming activities through the mathematical tools in use and the object of the activity. According to my analysis, the role of the teacher influenced object development through negotiations with the students and through that, also the tools in use. In the following subsections, I will take a closer look at how the role of the teacher influenced the object of the activity and the tools in use by reflecting on my key findings from all articles.

6.2 The influence of the role of the teacher on the object of the activity

As discussed in Article 3, the teacher has the opportunity to influence the object of the activity by negotiating with students. In this study, the teacher negotiated a common object together with the students, by mathematizing the students' already existing object with his suggestion.

Furthermore, after students succeeded with their task to drive the robot in a circle thus reaching their object, the teacher negotiated a new object with the students.

Mathematics then became the object of the activity when students started to find out why they needed to double their answer.

Thus, the negotiating role of the teacher influenced object development towards the expansion of the object. With the active role of the teacher, the object development was a transformative process. Students' original object to drive a circle transformed and mathematics became the object of the activity, thus finally expanded.

In the following, I will discuss how the role of the teacher influenced the mathematical tools in use, since the tools in use and the object development are in relation with each other in the activity system.

6.3 The influence of the role of the teacher on the mathematical tools in use

When comparing different sessions, I found out that mathematical tools were not systematically in use during the session where the teacher did not collaborate with the students during the activity design phase. During that session, the teacher did not participate in the object negotiation; the students decided upon their object. Thus, the teacher was not able to influence the mathematical tools in use. Moreover, the teacher could not help students with the needed programming tools because of his lacking programming skills.

During another session, the teacher negotiated the object together with the students through the use of suggestions. The teacher's mathematization of the common object enabled the systematical use of mathematical tools (Article 2). During this session, the teacher did not have any difficulties with the needed tools. Vice versa, the teacher's pedagogical and mathematical knowledge enabled fruitful activity development.

In summary, mathematics was linked with programming activities through the active role of the teacher during the object negotiation. Without the teacher's active role, the use of mathematical tools would have been limited.

6.4 The main argument

Based on the analysis of the articles, mathematics can be linked with programming activities through object development and the mathematical tools in use. In this study, the activity developed fruitfully with the mathematical tools in use when there was collaboration between the students and the teacher. The active role of the teacher enabled fruitful object development and transformation. The teacher was able to influence the tools in use, the object development, and collaboration between students through negotiations with students. The active relationships between the role of the teacher and object, tools and collaboration between students linked mathematics with programming activities. Thus, the main argument of this study is that the links between mathematics and programming activities have a transformative potential in mathematics education through the active and negotiating role of the teacher. In this subsection, I will discuss more the transformative potential of programming integration by reflecting on how the main argument of this study contributes to existing literature regarding the integration of digital technology into mathematics education.

Hoyles (2018) argued that digital technology has a transformational potential in mathematics education by opening different kinds of windows for students and, through that, changing the traditional practices in the classroom and providing outside of classroom connections. According to Hoyles (2015), one of the characteristic challenges in mathematics education is the invisibility of mathematics and, through that, the motivational factors; why learn the isolated subject of mathematics? Hoyles (2018) argued that the use of digital tools in mathematics provides an opportunity for students to become mathematical users instead of just learners.

However, Drijvers (2018), in his response to Hoyles (2018), called for some more detailed specifications for Hoyles' ideas about potential transformation in the mathematics classroom. As discussed in the introduction, this study corresponds with Drijvers (2018) suggestion to take a closer look at the potential links between mathematics and the use of digital tools. As Drijvers questioned Hoyles' suggestion about transformation practices in mathematics teaching and learning with three different claims, I will discuss these claims in the following by also reflecting on the findings of this study. The links between mathematics and integration of digital technology found in this study do not transform mathematics education, but show that programming integration has the potential for object transformations in classroom activities through the active role of the teacher.

6.4.1 Why to transform practices in mathematics education

Hoyles (2018) suggested that the transformations of classroom practices through the use of digital technology can enhance students' conceptual engagement. Drijvers (2018) responded that if students' conceptual engagement is seen as a measurable learning outcome, then the evidence of transformative power of integration of digital technology is weak and it is unclear why practices in teaching and learning mathematics should be transformed. This is in alignment with our findings in Article 1, where we argued that the results of students' outcomes could not be generalized, and individual tests do not provide enough information about successful programming integration. The studies measuring students' performance in mathematics after programming integration could not give any generalizable information about the usefulness of programming integration. The results improved for some of the groups, but for some of the groups there were no noticeable changes in student performance. Thus, in order to have a more detailed understanding of links between integration of digital technology and mathematics, this study has concentrated on students learning processes instead of learning results.

The fruitful activity development with the expansive object presented in this study shows the potential for transformations in mathematics education. The expansion presented in this study is not generalizable but shows the potential that programming integration can provide. These results could not have been measurable as students' outcomes, yet show that programming activities can be fruitfully linked with mathematics.

The need for this kind of transformations in mathematics education is visible in earlier studies discussing issues in a traditional mathematics classroom. As discussed in Chapter 2, in a traditional mathematics classroom, mathematics has a role as given tools and rules. The teacher gives students ready tasks, the *dead objects*, to solve (see Figure 8) (Engeström, 2008). The tasks are supposed to be solved with given mathematical tools and different fixed rules; thus, students often connect mathematics with different rules, procedures, and memorization (Albert & Kim, 2013; Bray & Tangney, 2017; Hoyles, 2016; Opheim & Simensen, 2017; Pietsch, 2009). The situation could be different from an object which is more alive. I will discuss the potential of more *alive* object in the following subsection.

6.4.2 Into what might mathematics teaching and learning be transformed?

Hoyles (2018) argued that introduction of digital technology can provide students with opportunities to use mathematics that is embedded in the digital technology and thus open new windows in mathematics education. Drijvers (2018) claimed that it is unclear how these kinds of opportunities can be utilized in the classroom and into what might teaching and learning be transformed. In their earlier study, Drijvers et al. (2010) have found that while the teachers wanted to concentrate more on the mathematics behind the digital technology in their teaching, the students were more focused on the technology itself.

The links between mathematics and programming activities were not self-evident in this study either. The process of mathematization was needed. Again, the process of mathematization was the result of the active role of the teacher during the object negotiation phase. And so, as discussed, the mathematized object enabled a fruitful activity development toward the expansion of the object. Due to the transformative object development, the role of mathematics in the classroom differed from the role of mathematics in a traditional classroom model (see Figure 8). While in a traditional mathematics classroom, mathematics has a stable role as given tools and rules, and it bases itself on the *dead object* (see Figure 8), in this study (see Figure 14), the role of mathematics was under development, it transformed, changed and expanded. The object of the activity transformed from *dead* to *alive*.

During the transformative process in this study, the teacher acted as a negotiator and guide instead of a lecturer. Certain turning points during the activity development and the students' and teacher's collective choices during the activity development influenced the students' learning processes. Thus, this kind of collective learning process cannot be predicted or measured with individual tests afterward.

In summary, in this study, mathematics was linked with programming activities, through the process of mathematization and the active role of the teacher. A response to Hoyles (2018) that was not self-evident. The links between mathematics and programming activities enabled a fruitful activity development with transformative object development. Due to object development, the role of mathematics was alive in the students' learning process unlike in a traditional learning process. This is one example of into what learning processes in mathematics can be transformed through programming integration as a response for Drijvers (2018).

6.4.3 By whom would the transformations be made?

Hoyles (2018) claimed that digital technology as a tool has the potential to transform teaching practices in mathematics classrooms. As such, Drijvers (2018) responded that digital technology as a tool as such could not make any transformations. The transformative potential of digital technology is constituted by how the tools are used or planned to be used by teachers or educational designers. According to earlier studies, teachers may as well use traditional teacher-led approaches regarding the integration of digital technology (Drijvers, 2018; McCulloch, Hollebrands, Lee, Harrison, & Mutlu, 2018) and no significant transformations are noticeable in the teaching practices. Drijvers et al. (2010) claimed that teachers' practices are quite stable with their certain regular habits and views on mathematics. In their study, they argued that when using teacher's instrumental orchestration as a theoretical framework, the orchestrations (i.e., the teachers' didactic and practical choices regarding technological tools in use and classroom activities) used by the teacher with regard to the integration of digital technology were close to traditional teaching practices. Thus, according to Drijvers et al. (2010), the potential transformations with digital technology are closer to *evolution*, rather than *revolution*.

According to earlier studies, the teacher's use of digital technology in mathematics education and the teacher's practices with digital technologies depend on their technological and pedagogical knowledge (Drijvers et al., 2014; Goos & Bennison, 2008; McCulloch et al., 2018; Wachira & Keengwe, 2011). Drijvers et al. (2014) highlighted that the lack of technological knowledge and skills might be a challenge for the teachers when integrating digital technology in a satisfying and useful way.

In this study, the teacher lacked the needed programming skills to guide students in certain circumstances in the entirely student-centered approach (Article 3). The role of the teacher in the object negotiation phases was a central component for the activity development. During another session, the teacher compensated his lack of programming skills with robust, pedagogical and mathematical knowledge. These findings indicate that programming tools have the potential to transform practices in mathematics classroom through the active the role of the teacher, teacher's knowledge and skills. Thus, I agree with Drijvers (2018) in saying that technological tools do not have the transformative power in the classroom alone.

In summary, based on my findings, I argue that programming integration has transformational potential in mathematics classrooms, but it is not self-evident. The arguments above do not indicate that programming integration automatically transforms learning and teaching practices in mathematics classrooms. However, as a contribution to the debate between Hoyles (2018) and Drijvers (2018), this study does show that programming integration has transformational potential in mathematics education. It is shown that during programming activities mathematics has a possibility to become the *alive* and transformative object of the activity. That is constituted by the active and negotiating role of the teacher, the teacher's knowledge and skills and other components during the activity development such as tools and object development.

6.5 Suggestions for future studies

This study has focused on the beginning of the programming integration, and it corresponds to the situation where the mathematics teacher does not have an extensive training in programming. I have shown that the integration of programming is still possible and there is a potential for fruitful learning sessions when the teacher and students collaborate. Based on this, I have introduced an example of how mathematics can be linked with programming activities, which worked at the beginning of the programming integration by motivating students and enhancing students' learning in mathematics; it is still unclear what happens over time. According to earlier studies, there is no convincing evidence that programming activities can enhance students' long-term motivation to learn mathematics (Article 1). The long-term results remain unclear also in this study. However, the example presented in this study could have the potential to provide holistically lasting benefits in mathematics education by transforming the object and the role of mathematics in the activities of the mathematics classroom. The usefulness of this example is constituted by many factors and the pressures from outside of the classroom.

Thus, in order to find out the usefulness of this kind of example, in the long run, it would be interesting to test this after the students and the teachers are more familiar with robots and programming. More widely, it would be interesting to test this example with research that is more extensive by discussing the effect of this model for potential transformations in mathematics education more broadly.

Furthermore, as discussed, the learning situation introduced above is not measurable with individual tests. However, teachers must evaluate the students' learning by giving them individual grades. How this is possible with the introduced example is a fascinating subject for future studies.

6.6 Possible implications for policy and practice

As the findings of this study show, programming tools alone do not make any fruitful changes in mathematics classrooms. The transformational potential of programming in mathematics education is constituted by the role of the teacher, his knowledge and skills. In this section, I will discuss the possible implications of the findings of this study for policy and practice.

Educational designers in the Nordic countries, Finland, Sweden, and Norway, integrated, or are planning to integrate, programming in the mathematics curriculum. However, as mentioned in the introduction, integration represents practical, everyday challenges. In order to discuss the possible implications of the findings of this study for policy and practice, I reflect the findings of this study with four everyday challenges presented in the introduction. These challenges were: (1) the links between mathematics and programming activities; (2) time; (3) the background of the teacher; and, (4) how programming influences students' learning. I will address these challenges separately in the following subsections.

6.6.1 Links between mathematics and programming activities

According to reports conducted by European Schoolnet, it is unclear how programming can be linked to different subject areas (Balanskat & Engelhardt, 2015; Bocconi et al., 2018) in mathematics. The findings of this study show that programming activities can be linked with curriculum mathematics. Even though the data collection for this study was not conducted in a mathematics classroom, the observations are transferable to the mathematics classroom because the activities support the mathematics curriculum with the development of problem-solving skills and the use of mathematical tools. The free rules in the classroom enable a fruitful activity development where the programming activities can be linked with the curriculum through the negotiating and active role of the teacher. The free activity development enables fruitful learning processes in mathematics. However, that is not self-evident and not predictable beforehand. Thus, there is a risk that these kinds of activities do not fit in highly detailed curriculums with specific goals.

However, the curriculum goals regarding links between mathematics and programming activities are not very restrictive, and connections between programming and mathematics are not specified in detail, at least in Finland and Sweden. Thus, the curriculum connections found in this study could serve well, for instance, in the mathematics curriculum of Finland and Sweden. In Finland, for instance, it is suggested to use student programs as a part of mathematics education (Opetushallitus, 2014). In Norway, when comparing these results with suggestions for curriculum reform in 2020, it seems that activities presented in this study suit the Norwegian way. It is stated in the suggestions in Norway that "Through programming, students can be more creative in approaching issues and gain the ability to explore connections that have not been possible to explore before." (Utdanningsdirektoratet, 2018).

6.6.2 Time

Programming as a very new curricular activity is assumed to take time and energy from other activities in the mathematics curriculum (Bocconi et al., 2018). As discussed, the links between programming activities and curriculum mathematics are possible. Thus, programming activities can be smoothly integrated with other activities in the classroom without wasting time. Thus, I would not argue that programming activities take time away from the teaching of mathematics. Vice versa, programming has the potential to bring valuable, out-of-classroom connections with otherwise isolated curriculum mathematics.

6.6.3 The background of the teacher

According to reports conducted by European Schoolnet, the role of a teacher may be challenged in programming integration, if the mathematics teacher does not have a relevant programming background (Bocconi et al., 2018). The teacher in this study did not have any degree in programming, and he lacked specific programming skills. Thus, he faced some challenges when students needed advice. However, he was able to compensate his lacking programming skills with strong pedagogical and mathematical knowledge. His pedagogical and mathematical knowledge was one of the critical elements in a successful robot integration as discussed in Article 3.

Based on the findings of this study, the teacher's knowledge and skills are essential for successful programming integration in mathematics education. Thus, it is not enough that schools get the needed tools; it is also important to allocate funds for in-service teacher training. This need was also visible in the findings regarding Norwegian teachers in TALIS (Teaching and Learning International Survey) conducted in several European countries in 2018 (Throndsen et al., 2019). Throndsen et al. (2019) reported that it is not enough that the teachers receive new equipment in the classroom. They also need advice on how to integrate digital technology into their teaching in an appropriate way.

Thus, it is crucial to focus and invest in both in-service and pre-service teacher training, regarding programming skills, but also regarding teacher pedagogical skills among programming activities.

6.6.4 How programming influences students' learning

According to reports conducted by European Schoolnet, it is unclear how programming influences the students' learning in mathematics (Balanskat & Engelhardt, 2015; Bocconi et al., 2018). This is in alignment with our findings from the literature review article (Article 1): it is unclear if programming activities improve students learning in mathematics.

According to earlier studies, working as a group and collaboration between students is usual in programming activities. However, when discussing the benefits of programming in mathematics education, student learning is measured with individual tests and grades (Article 1). One possible reason for this is that collective learning is challenging to measure. Another reason surely is that the education system and curriculum goals are based on individual grades and test results.

This study has discussed students' activities as collective processes, and learning is seen as a change in students' collective object within their collective activity according to CHAT (e.g. Engeström, 2005). As discussed in Article 2, the students' object expanded as a result of their fruitful activity development. Learning was visible through the students' enthusiasm to apply their learning in a new, broader situation.

However, the described free environment for object and activity development may be in a contradiction with pressures outside of the classroom. As I have discussed in the theory chapter, there is a possible contradiction between free and innovative activities with robots and pressures from outside of the classroom to produce good test results. A traditional classroom model indeed concentrates on the test results and grades. Students' collective learning described in this study probably could not have been measurable by an individual test at the end of the session. The collective learning with robots was not necessarily conscious, but it manifested in students' collective activities through object development. Learning was visible given the expansion of the students' ordinary object. So, even if this kind of collective learning might be unconscious and difficult to measure with traditional tests and grades, it is valuable learning for the students' future. When preparing students for the 2030-century, collaboration, communication, creativity, engagement, and critical thinking are highlighted (OECD, 2018). Learning mathematics through programming and robots also serves in the development of these skills. The question then becomes whether there is a need for transformations in the assessment practices in mathematics classrooms.

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Appendix 1.

Til elever og foresatte i XXXX ungdomsskole i valgfag «forsikring i praksis».

Forespørsel om tillatelse til undersøkelse, observasjon og video av elever i forbindelse med en doktoravhandling

"Programmering i matematikkundervisningen"

Bakgrunn og formål

Jeg er en stipendiat i matematikkdidaktikk ved Høgskolen i Østfold. Høsten 2017 skal jeg fullføre dette studie i XXXX ungdomsskole. Temaet er programmering i skolen. Formålet med studien er se på aktiviteten i klasserommet når elevene programmerer Lego Mindstorms og hvordan matematikk elevene anvender. Elevene fra valgfag «forskning i praksis» forespørres om at vara med.

Hva innebærer deltakelse i studien?
For å få best mulig datamaterialet til oppgaver ønsker jeg og observere undervisningen i valgfag leksjonene ca. 8 uker høst 2017. Jeg ønsker også å ta videodata i tre samlinger.

Hva skjer med informasjonen om deg?
Alle opplysninger som blir samlet inn bli behandlet konfidensielt, og ingen enkeltpersoner vil kunne gjenkjennes i prosjektoppgaven. Alle opptak slettes når oppgaven er ferdig, senest 31. Juli 2019. Prosjektet skal etter planen avsluttes 31. Juli 2019.

Frivillig deltakelse
Det er frivillig å delta i studien, og du kan når som helst trekke ditt samtykke uten å oppgi noen grunn. Dersom du trekker deg, vil alle opplysninger om deg bli anonymisert.

Jeg håper at dere finner dette interessant og ønsker å bli med på min forskning. Dersom du ønsker å delta eller har spørsmål til studien, kan du kontakte meg på telefon eller e-post.

Studien er meldt til Personvernombudet for forskning, NSD - Norsk senter for forskningsdata AS.

Mvh.
Sanna Forsström
Tel. +358 407 042 704
Epost: sanna.forsstrom@hiof.no

Samtykke til deltakelse i studien

Jeg har mottatt informasjon om studien, og er villig til å delta
Please fill out.

(Signert av foresatte, dato)

Appendix 2.

Sanna Forsström

Avdeling for lærerutdanning Høgskolen i Østfold

Remmen

1757 HALDEN

Vår dato: 09.06.2017 Vår ref: 54334 / 3 / BGH Deres dato: Deres ref:

TILBAKEMELDING PÅ MELDING OM BEHANDLING AV PERSONOPPLYSNINGER

Vi viser til melding om behandling av personopplysninger, mottatt 05.05.2017. Meldingen gjelder

prosjektet: *54334 The use of programming in mathematics education*
Behandlingsansvarlig Høgskolen i Østfold, ved institusjonens øverste leder
Daglig ansvarlig Sanna Forsström

Personvernombudet har vurdert prosjektet og finner at behandlingen av personopplysninger er meldepliktig i henhold til personopplysningsloven § 31. Behandlingen tilfredsstiller kravene i personopplysningsloven.

Personvernombudets vurdering forutsetter at prosjektet gjennomføres i tråd med opplysningene gitt i meldeskjemaet, korrespondanse med ombudet, ombudets kommentarer samt personopplysningsloven og helseregisterloven med forskrifter. Behandlingen av personopplysninger kan settes i gang.

Det gjøres oppmerksom på at det skal gis ny melding dersom behandlingen endres i forhold til de opplysninger som ligger til grunn for personvernombudets vurdering. Endringsmeldinger gis via et eget skjema, http://www.nsd.uib.no/personvernombud/meld_prosjekt/meld_endringer.html. Det skal også gis melding etter tre år dersom prosjektet fortsatt pågår. Meldinger skal skje skriftlig til ombudet.

Personvernombudet har lagt ut opplysninger om prosjektet i en offentlig database, http://pvo.nsd.no/prosjekt.

Personvernombudet vil ved prosjektets avslutning, 31.07.2019, rette en henvendelse angående status for behandlingen av personopplysninger.

Vennlig hilsen

Kjersti Haugstvedt

Belinda Gloppen Helle

Kontaktperson: Belinda Gloppen Helle tlf: 55 58 28 74

PART II: Articles

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A Literature Review Exploring the use of Programming in Mathematics Education

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Abstract. Programming is now included in mathematics curricula in several countries; thus, the purpose of this literature review is to determine the research-based justifications for these educational decisions. From a selection of relevant articles, 15 articles were identified and analyzed, each of which had varying study types, themes, and designs. Three themes from the studies were identified: the motivation to learn mathematics, student performance in mathematics, and the collaboration between students and the changed role of the teacher. It was found that in certain circumstances, including programming in mathematics education could improve student motivation to learn mathematics and improve student performance in mathematics. To gain a better understanding of the potential of programming in mathematics education, the entire collective learning process should be considered by discussing the roles of the teacher and the collaboration between students as part of these roles.

Keywords: mathematics education; programming; robots.

1. Introduction

We are facing the fourth industrial revolution, which is characterized by a range of new technologies that are fusing the physical, digital and biological worlds, influencing all disciplines, economies and industries (Schwab, 2017). According to Balanskat and Engelhardt (2015), in the future, many of today's students will be involved in developing technology, which is important for the society. Consequently, programming skills have become increasingly important core competencies for 21st-century skills and have become important in education policies seeking to adapt the education sector to meet future societal demands. Many countries have recognized that programming needs to be integrated into school curricula to equip students with skills, such as problem solving and

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logical thinking, which are important in today's digital society. The challenge for the education sector, therefore, is to give students the competencies to master and create their own digital technologies and to prepare them for the future; therefore, learning how to code and program in formal and non-formal education settings is vital.

As programming has come to be recognized as a basic skill for effectively participating in the digital world, there has been increasing interest during the past decade in introducing programming as a school subject (Grover & Pea, 2013). Programming is the process related to the development and implementation of instructions for computer programs so the computer can perform specific tasks, solve problems, and support human interactions. Therefore, programming generally requires programmers to have a knowledge of programming languages; expertise in subjects related to the development of specialized algorithms and logic; and the ability to analyze, understand, and solve problems by verifying algorithmic requirements and assessing the correctness and implementation (often referred to as coding) of the algorithm in a particular programming language. Because these processes have been linked to mathematical thinking, several European countries have claimed that since programming is related to the development of algorithmic thinking (Grover & Pea, 2013), it is an important skill for the digital society and the 21st-century skills of problem solving, creativity, and logical thinking. While there have been many different suggestions as to where programming might fit, there has been little consensus on how to include programming in school curricula (Grover & Pea, 2013), with the debate focusing on whether is part of Information and Communication Technology (ICT) or whether it should be integrated across the curriculum. Increasingly, schools have integrated programming into other subjects, mostly mathematics, using cross-curricular approaches (Balanskat & Engelhardt, 2015). Finland and Sweden, for example, have both integrated programming into mathematics (Bocconi, Chioccariello, & Earp, 2018; Opetushallitus, 2014; Skolverket, 2018) with the rationale that it fosters problem-solving and logical-thinking skills and motivates students to learn mathematics. Norway is planning to integrate programming in mathematics in the revised version of the curriculum in 2020 (Bocconi et al., 2018). According to Bocconi et al. (2018), further discussion is needed, however, on the ways in which programming can be linked with other subject areas and the degree to which it influences student achievement. Furthermore, a need exists for a discussion of the type of pedagogical solutions that are effective by considering concrete implementations of programming using a variety of tools and assessments. One relevant topic to consider regarding pedagogical solutions is the role of the teacher. When integrating programming with a mathematics curriculum, the role of the teacher may become challenging because the mathematics teacher may not have previous knowledge of programming.

Using programming in mathematics education is not a new concept. As early as 1980, Papert (1980), who associated learning through programming with Piaget's constructivist learning theory, developed a Logo environment that required students to program a computer to steer a turtle on a computer screen, with the

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intention of providing a different environment for learning mathematics and motivating students to engage with mathematics.

Based on Papert's Logo environment, Yelland (1995) examined "the potential of Logo to act as a mathematical environment" (p. 853) in a review article examining the relationship between cognitive gains, problem-solving, and social interaction skills in student mathematics achievements in dozens of quantitative and qualitative studies. Yelland found that there had been varying results regarding the cognitive gains in problem-solving and mathematics achievements. Yelland (1995) noted, however, that Logo was a useful learning environment for students from both individual and group perspectives and that it was a helpful way for researchers to understand the thinking and learning processes since Logo gave students the opportunity to explore mathematics in a meaningful way. However, the outcomes were contradictory. Although some studies showed evidence of the positive impact of Logo's inclusion in mathematics, others failed to detect any differences in the students' problem-solving skills and mathematics achievements after completing the Logo programming projects.

Since Yelland's review in 1995, some significant technological developments have resulted in a number of different programming environments for classroom use, such as Scratch, and programmable robots, such as Lego Mindstorms. Benitti's (2012) literature review, "Exploring the educational potential of robotics in schools: a systematic review," examined ten quantitative studies on the educational potential of robotics and concluded that even though some studies had found no differences in the students' learning, robots were useful in understanding science, technology, engineering, and mathematics (STEM) concepts. For example, positive mathematics achievements were found for certain topics, such as circle geometry and degrees, fractions, and proportions, and for certain groups of students, such as those with average scores or in certain grades (Benitti, 2012; Lindh & Holgersson, 2007; Nugent, Barker, & Grandgenett, 2008). However, the studies did not find any improvements in student achievement for certain topics, grades, or groups of students, such those with high and low scores (Benitti, 2012; Lindh & Holgersson, 2007), and some of the results for student problem-solving skill developments were ambivalent. Based on various mathematics and problem-solving tests, such as pre- and post-tests with control groups (Benitti, 2012; Hussain, Lindh, & Shukur, 2006; Lindh & Holgersson, 2007), some studies (Nugent, Barker, Grandgenett & Adamchuk, 2009) found positive results, while others (Benitti, 2012; Hussain et al., 2006) detected no improvements.

In a more recent review, "How have robots supported STEM teaching?,"Benitti and Spolaôr (2017, p. 104) analyzed 60 studies from 2013 to 2016 and found that technology and engineering education appeared to benefit most from the inclusion of robotics; however, the potential use of robots in mathematics education was seen as a support tool. In general, robotics education tended to be used as part of extracurricular activities (57% of studies) or out-of-school activities (25% of studies) rather than as part of general curricular activities (18%

of studies). Benitti and Spolaôr argued that one reason for this phenomenon may have been the teacher's poor or inaccurate knowledge of robots; however, this argument requires further evidence. Regardless, the most often-observed skills development associated with robotics was problem-solving and teamwork skills; however, even though teamwork has been commonly connected with robotics education, Benitti and Spolaôr (2017) reported that only three of the 60 studies included collaborative learning theories, all of which had been out-of-school activities.

Overall, in the current climate, Yelland's (1995) review is now out of date, and Benitti (2012) and Benitti and Spolaôr (2017) only examined robots rather than programming and only discussed mathematics as part of their reviews. Because programming has become or is becoming compulsory in the education of students aged 6–16 in many countries and because some countries have already integrated programming into mathematics curricula (Balanskat & Engelhardt, 2015), the contribution of this paper is to provide an updated review of studies on the use of programming and robots in mathematics education for students aged 6–16.

The main aim of this article is to answer the following question: What is the educational potential of programming in mathematics education?

2. Methodology

To answer our research question, we conducted a literature review with a systematic search and selection of articles. The implementation of the search, selections of the articles and design and theme of the selected articles are presented in this section.

2.1 Planning and conducting the review

To answer the research question, studies that examined programming, coding or robots and mathematics education for students aged 6–16 were searched for using the following search terms: teach*, learn*, education*,robot*, Lego, programming, coding, school, K-12 and mathematics*.

The search in the databases employed the Boolean operators AND, OR, and NOT and used the search terms in the keywords, topics, titles, and abstracts of the articles. For the final search, five databases were used: IEEE XPLORE, ScienceDirect, Education Resource Information Center (ERIC), Wilson Education, and the Web of Science. Articles written in peer-reviewed English language journals published between 1995 and 2018 were searched to identify all articles written since Yelland's 1995 review.

The initial search identified several articles that were outside the scope of interest. We therefore developed five additional exclusion criteria:

1. The article does not deal with programming, coding, or robots.
2. The article does not deal with education.
3. The article does not deal with mathematics.
4. The article does not deal with students aged 6–16.

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5. The article does not outline a research design and research questions (these articles are often experience based).

Initially, both authors independently read the titles, article abstracts, and whole articles to determine which to select for further reading. After careful independent second readings of the whole articles and following thorough discussion, the relevant articles for this review were chosen. The articles that were not relevant dealt with the technical details of robots or had no empirical data from the schools. Articles that only focused on ICT education without reference to mathematics education and articles analyzing activities outside the classroom, such as summer camps, were also omitted.

Table 1 shows the articles identified in the search, the number selected for further reading, and those selected for the final analysis.

Table 1: Article selections

Database	Articles	1st selection	2nd selection
IEEE XPLORE	166	11	0
Web of Science	197	11	7(4 duplicates with ERIC)
ERIC	143	28	8
ScienceDirect	261	3	3 (2 duplicates with ERIC)
Wilson Education	150	20	11 (8 duplicates with ERIC)
Total	917	73	15

2.2 Design and theme of the studies

After careful reading, we identified and compared the themes and designs of the studies and selected four dominant themes for further discussions on the educational potential of programming in a mathematics education: students' motivation to learn mathematics, students' performance in mathematics, and collaboration between students and the changed role of the teacher. Table 2 shows the themes and designs for the 15 relevant articles identified for the literature review. However, it was difficult to determine the design for some studies.

Table 2: Articles and the robot type or programming language, topics, student ages, and data analysis methods.

Article	Robot type/ software	Topic	Age	Methods	Duration of the data gathering
Lambic (2011)	C++ Builder	The motivation of students to learn mathematics	13–19	114 participants, pre- and post-questionnaires	9x45 min
Moreno-León, Robles, and Román-González (2016)	Scratch	Impact of introducing programming in several subjects: 1. academic performance, 2. student perception, 3. assessment of projects with Scratch	11–12	129 students, experimental and control groups, pre- and post-tests	8 weeks
Taylor, Harlow, and Forret (2010)	Scratch	Potential of Scratch to enhance mathematical and technological thinking	9–10	60 students, observations, video recordings, teachers blogs, teacher interviews	
Lindh and Holgersson (2007)	Lego Mindstorms	Pupils learning', learning context/classroom environment, the role of the teacher	11–12 15–16	322 students, experimental and control groups, observations, interview, inquiry	12x8h (12 months, 2h/week)
Hussain, Lindh, and Shukur (2006)	Lego Mindstorms	Pupils' learning, learning context/classroom environment, the role of the teacher	11–12 15–16	322 students, experimental and control groups, observations, interview, inquiry	12x8h (12 months, 2h/week)
Khasawneh (2009)	Logo Programming	Student achievement, correlation between achievement in Logo programming and school mathematics achievement. Problem-solving ability	12–13	228 students, post-test	15x45 min

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Bartolini Bussi and Baccaglini-Frank (2015)	Bee-bot	Semiotic potential of bee-bot with learning of rectangles	6–7	18 students, observations, photos, graphical productions, video recordings	4 months (15 sessions)
Falloon (2016)	Scratch Jnr. on the iPad	General thinking skills	5–6	32 students, audio capture on iPads	5x(25–40)min
Ardito, Mosley, and Scollins (2014)	Lego Mindstorms, Turtle Art	Student mathematical understanding, student experiences and practice in problemsolving and collaboration	11–12	Teacher interviews, classroom observations, State exam	14 weeks
Ke (2014)	Scratch	Student participant attitudes toward mathematics before and after game-making activities, mathematical thinking	13–16	64 students, pre- and post-inventory	6x1h
Leonard et al. (2016)	Lego Mindstorms	STEM attitudes, computational thinking, self-efficacy in technology	13–16	124 students, pre- and post-survey	60 h
Barak and Assal (2018)	Robots	Students' working patterns, achievements and difficulties in learning a STEM-oriented robotics course, impact on student motivation to learn STEM subjects	13–14	32 students, pre- and post-questionnaires	15x90 min
Sinclair and Patterson (2018)	Dynamic geometry environments	How computational thinking and mathematical thinking relate?	14–16	Student sketches	2 years

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Husain, Kamal, Ibrahim, Huddin, and Alim (2017)	Scratch	Mathematical thinking skills, problem solving	10–12	95 students, pre- and post-tests	1,5 day
La Paglia, La Cascia, Francomano, and La Barbera (2017)	Lego Mindstorms	Mathematical and metacognitive skills, reasoning and problem-solving capabilities, attitudes toward mathematics	10–12	60 students, experimental- and control groups, questionnaires	10x3h

3. Results

We will discuss four different dominant themes from studies on the educational potential of programming in mathematics education: the students' motivation to learn mathematics, the students' performance in mathematics, and the collaboration between students and the changed role of the teacher. In this section, these four themes are separately discussed. Articles that mentioned student interest, attitudes, mind-set, contribution, engagement, joy or happiness in learning mathematics are discussed under the motivation category. Student performance refers to students' academic achievements as measured quantitatively in test results as well as improvements in students' mathematical thinking and problem-solving skills based on similar quantitative and qualitative data, such as classroom observations, teacher and student interviews, and teacher and researcher blogs. Increased collaboration between students and the changed role of the teacher are discussed as potential in students' learning processes in mathematics.

3.1 Student motivation to learn mathematics

In this section, we review how programming and robots affected the student motivation to learn mathematics. Five articles discussed students' motivation or attitudes to learning mathematics or their interest in STEM topics.

Regarding our analysis, programming provides an opportunity for students to connect mathematics to real life in a new way; thus, programming has the potential to influence their attitudes toward mathematics (Ke, 2014; Lambic, 2011). La Paglia et al. (2017) discovered that using Lego Mindstorm robots improved their attitudes towards mathematics. Barak and Assal (2018) found no significant change in students' attitudes toward STEM topics in tests before and after their activities with Lego Mindstorm robots because students' motivation was already quite high before the programming activities. In Leonard et al. (2016), no significant changes were observed in either students' STEM attitudes or interest in STEM careers during the intervention period.

Regardless, although some studies found that programming and the use of robots motivated students or improved their attitude toward mathematics, generalizing these findings was not possible. First, each study was conducted outside the mathematics classroom as an extracurricular activity or as part of a science or technology education. Leonard et al. (2016) focused only on underserved and underrepresented students. In Ke (2014), only 20% of the participants were native, and the student age range in Lambic (2011) was wider than other studies. In all studies, the student arrangements were different from regular classroom activities since they were extracurricular activities or there were additional people in the classroom, which could affect student motivation. Second, no evidence was provided on what happened over time, especially when the programming activities were integrated into normal classroom routines. Therefore, it was not possible to assess whether programming enhanced motivation. Furthermore, a comparison of the study designs showed that the motivation to learn mathematics was examined as part of a broader study that also examined other STEM subjects, computational thinking, and metacognitive skills.

3.2 Student performance in mathematics

Five of the articles quantitatively examined student learning by measuring changes in the students' grades or test results. All of these studies had something positive to say about students' learning mathematics after the test periods with programming. Even if the test results did not show any improvements in some of the cases, each study reported positive improvements for some groups. Moreno-León et al. (2016) found the use of Scratch to have accelerated the mathematics learning of the experimental group; however, the effect was larger for social studies. Lindh and Holgersson (2007) and Hussain et al. (2006) were based on the same study and data. It was found that the Lego Mindstorms robot activities were possibly useful for some groups; however, there was no overall effect. While the fifth-grade students' mathematics results improved after the Lego training, there were no changes for the ninth-grade students and no noticeable improvements in problem-solving skills in either the fifth- or ninth-grade students. The teacher in Ardito et al. (2014) found that the students showed improvements in some mathematical topics, such as area and circumference, the quantitative data did not support these findings when comparisons were made across the whole state. However, other data indicated that the students had better results in problem solving and logical thinking. Khasawneh (2009) compared student mathematics achievements with student Logo programming achievements. A positive but low correlation was found in seventh-grade students.

Even if the studies brought out some positive effects on student performance, the results are not generalizable. First, the improvement was only visible in certain groups. Second, the comparison concentrated on different components in different studies. For instance, the comparison in Khasawneh (2009) focused on programming achievements. Ardito et al. (2014), Lindh and Holgersson (2007), and Husain et al. (2017) used tests that corresponded to the national tests in the

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countries in which the studies were conducted. Third, improvement was shown only in certain mathematical topics because some of the topics were better suited to programming activities. According to this review, programming tasks are often connected to geometry. The connection between circle geometry and robotics activities is natural since in some cases, the students needed to determine the circumference of the robot wheels when programming the robot to move a certain distance (e.g. Leonard et al., 2016). Additionally, the activities with Scratch often are connected with geometry. To use plane geometry, such as squares, triangles, circles and angles, is common in the Scratch activities (e.g. Moreno-León et al., 2016).

3.3. Collaboration between students and the role of the teacher

Different pedagogical practices, such as collaboration between students and the role of the teacher in the classroom, were part of the discussion in the articles. Student collaboration was widely used in programming and robot-based activities. Based on the articles, the collaboration between students depends on several different ways on the role of the teacher in the classroom. First, the role of the teacher as a support and guide instead of as a lecturer enables students to solve problems in groups (Taylor et al., 2010). Second, the teacher acts as a conflict solver in the classroom. The teacher needs to be present with arguments when students face challenges and their collaboration breaks down. The teacher is able to make the collaboration work again by discussing the problems with the students (Hussain et al., 2006; Lindh & Holgersson, 2007). Third, the classroom climate created by the teacher is important for students' collaboration. According to Taylor et al. (2010), the classroom culture, in which students respect others' views and listen to each other, increases the collaboration among students and changes in their ordinary roles. The free environment in the classroom provides opportunities for students to adopt group roles that are different from that of their mathematics group's ordinary lessons. Students who are normally categorized with low ability can gain the opportunity to lead the group and come out with sophisticated mathematical ideas.

Because the role of the teacher in the classroom affects student collaboration, the collaboration between students in programming activities affects their learning. Through collaboration and knowledge sharing, students gain an opportunity to learn from each other (Falloon, 2016; Taylor et al., 2010). Students share their knowledge in and among the groups (Barak & Assal, 2018; Hussain et al., 2006). Knowledge sharing even affects students' choices in their problem-solving strategies. As Hussain et al. (2006, p. 188) stated:

> *"One way to learn by children is by a "trial-and-error method." Another way is more "cooperative": by asking their fellow workers. Alternatively, they ask another pupil in the class that is considered to know the material much better than oneself."*

The articles reported a strong cohesion in the student groups; even so, students saw themselves as a group instead of as individuals who are conducting tasks by viewing their achievements as the group's achievements (Ardito et al., 2014). Barak and Assal (2018) reported that students' success and achievements as a group provided more valuable feedback than the teacher's feedback for the students. Regardless, learning processes with programming activities depend on the choices that students make as a group during their problem-solving activities (Taylor et al., 2010); thus, learning using programming activities cannot be predicted beforehand.

No deeper discussions occurred on the effect of collaboration, especially on students' mathematics learning. Furthermore, even if studies discuss collaboration as an important factor in students' learning, the learning is viewed in most studies as a change in individual knowledge instead of something that the group achieves as a group through collaboration. As in Khasawneh (2009, p. 623), students work individually only because of the assessment: "Often students work in groups in order to cover the turtle activities in the textbook. In the meanwhile they work individually for the purpose of assessment."

4. Conclusion

The fourth industrial revolution will create both opportunities and challenges. The digital technologies merge with physical, biological and economic systems. In the long term, this will create upheavals for all industries and technologies. We have to adapt these changes, and we must understand new technology and acquire skills such as critical thinking, computational thinking and interdisciplinary to handle these changes. The schools have a social mandate. This mission starts with the individual student, who is to acquire knowledge, skills and competencies, and educate and mould students to become citizens who will support and continue the society. Therefore, we need to adapt new digital technologies in schools. In addition, we need to know more about how we can integrate such technology in a school environment, as for instance, the use of robots in teaching mathematics programming. Education is a key arena for using and understanding digital information and programming in society. Education offers an extraordinary opportunity for developing programming skills (Balanskat & Engelhardt, 2015). Simultaneously, programming is transforming education. Programming is not only an educational tool but also creates new ways of learning and understanding knowledge. Therefore, policies for programming in education are crucial, as how policies are developed in educational practices. In general, there is agreement in policy that programming is important. As this literature review showed, sparse research exists on the educational potential of programming in a mathematics education. On the one hand, most of these articles drew out results showing better performance in mathematics and higher motivation to learn mathematics. On the other hand, the generalizability of these result is less clear. Most European countries face the situation, in which programming is included in a mathematics education (Balanskat & Engelhardt, 2015). However, we call for more research and research-based arguments in the policy for including programming in a mathematics education. A need exists for a better understanding of how programming is politically conceptualized and how these conceptualizations constitute educational practice.

There appear to be two ways in which countries could introduce programming into the curriculum: as a separate subject (such as technology or computing) or via integration into existing subjects. With regard to integration, since programming has most often been linked to mathematical thinking, there has been a tendency in several European countries to include programming to develop algorithmic thinking (Grover & Pea, 2013).While there is little doubt that programming skills are important and that they will become more important in the future, firmer guidelines are required as to the role programming has in school and the role it could play in mathematics education. The idea that programming could be helpful in mathematics education was first raised by Papert in the 1980s using the LOGO programming language. During the 1980s, there was great enthusiasm and confidence that LOGO and similar programming languages would radically reform mathematics teaching in primary schools; however, the results from mainstream implementations did not entirely live up to expectations (Misfeldt & Ejsing-Duun, 2015).

Because there have been strong moves to associate programming and mathematics, there is a need for these associations to be better reflected in the research literature. This literature review focused on programming and robots in mathematics education. The aim was to map existing research examining the use of programming in mathematics education to determine whether there was sufficient evidence to justify the integration of programming into mathematics curriculum and to identify areas for further research. In all, 15 selected articles were analyzed to determine the educational potential of programming in a mathematics curriculum. The characteristic themes discovered were increasing student motivation to learn mathematics, improving mathematics performance, and increasing collaborations with different types of teacher roles.

This study concentrates only on studies discussing programming and mathematics education. The limitation of this study is that it does not consider programming in a broader educational perspective, for instance, in other STEM subjects. We are also aware of several studies conducted that have been connected to after school programs and summer camps (especially in USA). Even though they provide interesting information about programming education, we did not consider them as a part of this review about mathematics education. Furthermore, we concentrated on compulsory school education and did not include studies discussing upper secondary education or pre-school education.

As Papert (1980) suggested in 1980, programming has potential in a mathematics education. Programming and robots provide a real-life connection in a mathematics education, which is an important factor in motivating students (e.g. Ke, 2014; Leonard et al., 2016). This potential is important in a mathematics education, which otherwise is experienced as quite an isolated school subject. A typical issue in a mathematics education is that students do not understand the purpose for their learning (e.g. Lambic, 2011).

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According to our analysis, at least the geometry part of the curriculum has a natural connection with programming activities. Much of the research on connecting programming to mathematics focused on geometry. We call for more research that connects programming with other fields in mathematics. Programming is related to the development of algorithmic thinking (Grover & Pea, 2013), with the rationale that it fosters problem-solving and logical-thinking skills and motivates students to learn mathematics. If students are given the opportunity to develop such abilities, they must use programming in subjects other than geometry. Regardless, programming activities provide opportunities to make connections with the mathematics curriculum. Programming activities can be connected to curriculum mathematics as least in geometry, but not necessarily in a traditionally prescriptive manner. The curriculum connection depends on the collective choices that students make during their problem-solving activities in programming. Thus, the connection with a mathematics curriculum cannot be predicted.

The potential for programming in a mathematics education make it well suited for mathematics, and the political decision to integrate programming in a mathematics curriculum can be justified. The concrete benefits of programming in a mathematics education depend on many factors that should be considered along with the integration. Programming activities and mathematics learning through these activities do not correspond to traditional learning situations in the classroom. Students’ learning processes with programming are often collaborative, and the teacher plays a different role than normal. The most commonly used learning theories in the studies were constructivist or social constructivist learning theories (e.g. Benitti & Spolaôr, 2017). To gain a greater understanding of the potential for programming in a mathematics education, the entire learning process should be considered by viewing learning as a collaborative process of the entire group instead of viewing learning only as an individual cognitive process or a socio-cognitive process. To consider the collective learning of the entire group or class instead of individual learning by analyzing interactions among students, the teacher and the programming tools can provide valuable information in addition to the current knowledge of the usefulness of programming in a mathematics education.

Furthermore, regarding pedagogical practices in the classroom, the role of the teacher is worth consideration because ordinary mathematics teachers will be required to teach programming. Although integrating programming in a mathematics education is a political decision, a comprehensive discussion on required competencies for mathematics teachers is needed. While programming is integrated in the mathematics curriculum, the highly considerable discussion is to also integrate programming to the pre service and in service teacher education curriculum.

Based on our conclusions, our suggestion for future studies is to consider students collective learning processes in mathematics through programming activities, by also discussing the influence of the role of the teacher in students learning processes.

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Learning mathematics through activities with robots

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Abstract

There are several countries that integrate programming into their mathematics curricula, thereby making robotics an interesting aspect of mathematics education. However, the benefits of using robotics for mathematics education are still unclear. This article addresses the use of mathematical tools with robot-based problem-solving activities by discussing how mathematical tools are used in robot-based activities. This ethnographic intervention study took place in one secondary school in Norway as a part of an elective class in which videotaped data were gathered by observing the activities of a group of two or three students using Lego Mindstorm robots during an eight-week period. Through the use of activity system analysis in Cultural Historical Activity Theory, the analysis found that students use different kinds of mathematical tools. Furthermore, mathematics can change its role from instrumental tool to object, that is, to an integrated aspect of the purpose of the activity.

Keywords: Robots, mathematics education, cultural historical activity theory, activity system analysis, mathematical tools, object of activity

Introduction

Education systems in various countries are integrating the teaching of programming into their curricula in a variety of ways, by including general information and communications technology courses and by integrating programming into individual subjects. Nordic countries such as Finland, Sweden, and Norway have integrated or are planning to integrate programming into the mathematics curriculum. A pedagogical discussion regarding the merits of integrating programming in a cross-curricular approach (Balanskat & Engelhardt, 2015) is necessary. There is a need for research-based knowledge on issues such as how programming can be linked with differing subject areas, how programming influences students' learning, and the interplay between pedagogical approaches to different kinds of programming with different kinds of tools and assessments (Balanskat & Engelhardt, 2015; Bocconi, Chioccariello, & Earp, 2018).

Currently, there are dozens of different robots and toolkits suitable for educational use (Karim, Lemaignan, & Mondada, 2015), with Lego Mindstorm robots being the most widely studied (Benitti & Spolaôr, 2017). In classrooms, students can steer and control Lego Mindstorm robots by programming motors with the help of a variety of pieces, sensors, and blocks (Savard & Freiman, 2016). How curriculum-related mathematics in robot-based activities is used is unclear. Savard and Highfield (2015) argued that even teachers cannot associate the mathematics used by students in robot-based activities with curriculum-related mathematics. According to Savard and Freiman (2016), students do not design the use of mathematical tools during the problem-solving activities with robots but concentrate instead on digital design.

The aim of this article is to contribute to pedagogical discussions regarding programming and robotics in mathematics education by taking a closer look at the use of mathematical tools in students' collective activities with robots. We achieve this by analyzing students' activities with Lego Mindstorm robots, drawing on Engeströms' (1987) Cultural Historical Activity Theory (CHAT), which is well suited for analyses of tool-mediated collective activities. In the perspective of CHAT, the use of tools is dependent on the object of the activity. The component of the object has a special and central role in CHAT. The object of the activity is understood in CHAT as a goal, motive, drive, direction or purpose, which subjects of activity aim collectively. With robots the object of the activity could for instance be, to program the robot to drive a certain path.

A more detailed use of mathematics in students' collective activities with robots is discussed by answering the following question: What is the relationship between mathematical tools and objects in robot-based collective student learning activities in secondary education?

In our earlier article, we have addressed the role of the teacher in students learning processes with robots. We discussed the relationship between role of the teacher and other components in students' activity development (Forsström, 2019). We found out that the role of the teacher in the beginning of the activity development influences the object (drive, direction and purpose) of activity and mathematical tools in use. However, the article did not discuss the activity development in mathematical tools mediated activity. Thus, this article concentrates on the relationship between mathematical tools and object (drive, direction and purpose) of the activity during activity development.

We want to analyze how the use of mathematical tools develops in situations in which students work in collaboration on a relatively open-ended task. In these situations, students are not obligated to use mathematics but might find it useful in the process of their activities and tasks. Furthermore, in this article, our interest is not individual, cognitive learning but learning processes in collective interaction and activity. In particular, we want to analyze how a group uses tools and how the group negotiates the object of the activity, that is, the purpose and motivation of their project. Learning is seen as a collective, transformative, and expansive process (Engeström, 1987; Kaptelinin & Nardi, 2006).

The robot activity of one group of students, aged 12–13, took place during an eight-week period, and was chosen as the unit of analysis. The data material was gathered through video recording and field notes based on observation. The evolving interaction between the human actors, robot, and mathematical tools was at the centre of attention. A micro-strategy allowed a detailed analysis of how mathematical tools are used in different manners and how the drive, purpose, motivation, and direction of the activity can be developed and changed.

Following this introduction is a review of the central literature discussing learning opportunities through activities with robots. A section on the theoretical framework of this study, CHAT, is provided next. In the methodology section, the research strategy, sampling constitution of data, and strategies of analysis are discussed. The principal section is partly the analysis of the use of tools and partly object development and expansion. The final section discusses how these findings contribute to existing literature.

Literature review

It is unclear how robots serve the curriculum in practice (Alimisis, 2013). Review articles, such as Alimisis (2013), Benitti and Spolaôr (2017), and Karim (2015), discussing the educational benefits of robotics, in general, have revealed a need for studies discussing the curriculum connection with robotics. Several studies considering robots in education have been conducted as part of an out-of-school activity or as an extracurricular activity (Benitti & Spolaôr, 2017).

Problem-solving activities with robots offer a different kind of learning environment in mathematics education by providing the opportunity to use mathematics in practice (Ardito, Mosley, & Scollins, 2014; Barak & Assal, 2018). The educational benefits of robotics in mathematics education are still unclear. Quantitatively, Lindh and Holgersson (2007) found that some groups of students improved their results in mathematics tests after training with Lego Mindstorm robots but with some of the groups, no improvement was noticed. The post-test results were compared with the pre-test results.

Qualitatively, Barak and Assal (2018) argued that, even if robotics can provide an informal and innovative learning environment and enrich mathematics learning by providing mathematics in action, it cannot substitute for systematic and formal mathematics teaching because of its informal nature. However, Bartolini Bussi and Baccaglini-Frank (2015) found out that the first grade students connected their informal activities with bee-bot-robots (programmable toy that resembles a bee) with formal mathematics concept of square. The students programmed the robot to drive an O-letter path. Because the robot turns only 90 degrees at time, the students called the path to “squarized O”, which consists of four right angles. This is an example how young students connected a formal mathematics concept in the informal activities with robots. Bartollini Bussi and Baccaglini-Frank (2015) argued that activities with robots might have the potential to open also other formal mathematical meanings for the students.

Savard and Freiman (2016) found that students used mostly a trial-and-error strategy in problem-solving activities with robots. Students often started with digital contexts without creating any design regarding the use of mathematics; mathematical tools were mostly in use through the trial-and-error strategy. Savard and Freiman (2016) argued that trial and error worked well in solving programming problems with robots, but it also acted as an obstacle to students in acquiring greater understanding in mathematics because students could not detect a source of error that they made within the mathematical context.

Large, quantitative studies, such as Lindh and Holgersson (2007), understand and assess learning as an individual change in a subject's knowledge. Students might solve problems in groups, but the tests and grades are individual. By contrast, Savard and Freiman (2016) used a sociocultural approach to gain understanding in the emerging mathematical reasoning. They conducted in-depth investigation regarding students' learning processes to acquire a better understanding of the complexity of assessing students' learning in mathematics through the activities with robots. On the issue of learning, they argued, "Knowing that students successfully performed the task is not enough: knowing which concepts and processes they used gives more information to position them within their learning process" (Savard and Freiman 2016, p. 109).

This means that investigating the effects of specific methodological approaches in education is not sufficient. Understanding the complexities and processes of learning as participation in collective activities is vital. In this article, we argue that different uses of mathematical tools and different objects in the activity give rise to very different learning processes and possibilities.

Theoretical framework

To analyze the relationship between mathematical tools in use and objects in robot-based activities, we examine students' learning processes with robots by drawing on CHAT, an analytical framework offering a reservoir of concepts, possible relations, and processes. The framework as a whole understands human action as social activities, and the analytical reservoir enables the analyses of the different aspects of and processes in and between activities. In this paper, we analyze the interaction of the group as an activity. The group consists of the students, the teacher, and the robot.

CHAT enables the analysis of interaction, that is, the interactive processes in the group activity. Furthermore, CHAT assists in understanding learning and change in the group as mediated by tools (Engeström, 2005). Having access to constructive tools and knowing how to use them are important in learning processes. Furthermore, any activity is constantly changing, developing, and shaping itself, and the activity system analysis in CHAT enables seeing the effects of various components, such as tools and objects, for that development (Engeström, 1987).

In Engeström’s (1987) activity system analysis (Figure 1), seven components, namely, subject, object, tool, rules, community, division of labor, and outcome, are connected. The components are listed in table 1.

Fig. 1 Activity system developed by Engeström (1987, p.78).

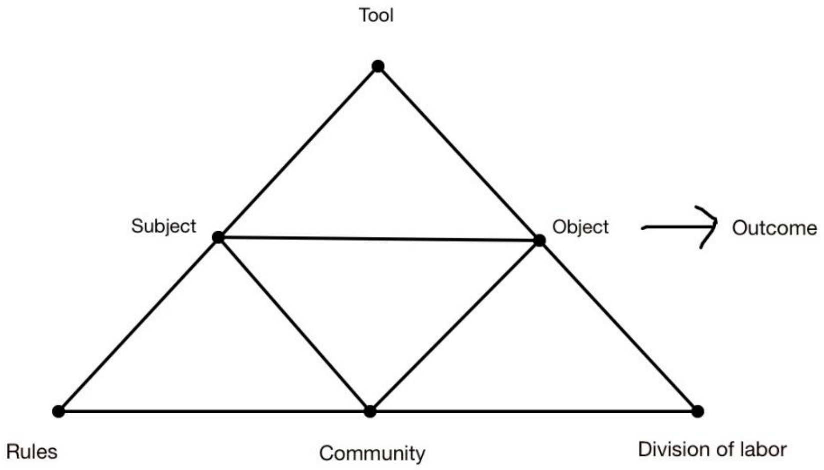


Table 1 Definitions of the different components in the activity system analysis

Component	Definition/meaning	Examples from this study
Subject	Individual or group of people engaging in the activity (Yamagata-Lynch, 2010)	Acting students and teacher
Object	Driving force in the activity (motive and goal) (Engeström, 1987)	Fulfill a task with the robot
Tool	Instrument mediating the activity (Engeström, 1987)	Robot, computer, mathematical tools, programming (coding)
Rules	Regulations relevant to the activity (Yamagata-Lynch, 2010)	Task assignment and rules from the mathematics classroom
Community	Social group the subject belongs to during the activity (Yamagata-Lynch, 2010)	Entire class of students and teacher
Division of labor	How the tasks are shared during the activity (Yamagata-Lynch, 2010)	Collaboration between students and the role of the teacher
Outcome	Result of the activity (Yamagata-Lynch, 2010)	Robot drives a track as programmed

Object of the activity

In the activity system analysis, activities are motivated and led by objects, an activity is always object-orientated (Engeström, 1987). In this context, the object determines the activity, and the activity is recognized and distinguished from other activities by its object. The concept of object has a special definition in CHAT as a collective goal, motive, direction or driving force in the activity (Engeström, 1987; Roth & Radford, 2011). Thus, the definition of object in CHAT differs from traditional everyday understanding about word object as a material thing or item.

Through a division of labour, subjects in the activity work collectively towards the same object (Engeström, 1987). The participants can manipulate and transform the shared objects, which can be a material or nonmaterial such as a plan or common idea. The object can also be changed during the activity (Kuutti, 1996). The subject aims towards the objects through tools (Engeström, 1987).

Tools

The subject relates to the object through the use of various tools. The use of tools depends on the objects of the activity (Engeström, 1987). The tools of an activity can be a material, such as computers and robots, or nonmaterial, such as rules, recipes, stories, and narratives. A language can be seen as a tool that enables communication. Mathematics is also a language (Ryan & Williams, 2007). A programming language enables communication with robots.

The activity is always tool-mediated and collective. Even though it appears that an individual has a direct contact with the object, there is always a connection with other individuals at least through some cultural tools such as gestures, pictures, or words. As activities are always collective, tools are also the results of the collective activities. The cultural tools, which enable collective activities related to other individuals, are the results of human beings’ collective life activities in practice (Engeström, 1987). Mathematics is a cultural tool, created over time by human beings. Different kind of mathematical tools can be, for instance, different formulas, algorithms, proportions, functions, and graphical models.

Often the use of tools is unconscious (Engeström, 1987). The focus can temporarily be on a tool, for example, when robots do not act as desired, and the students focus on the robots. However, this can only be a temporary state. Tools are not objects of the activity (Engeström, 1987).

Expansive learning

Unlike traditional learning theories, CHAT links learning with social transformations by linking individuals with social structures (Engeström, 2005). Learning is seen more as a long-lasting collective and expansive process than an individual result. The focus is on object development:

> Traditionally we expect that learning is manifested as changes in the subject, i.e., in the behavior and cognition of the learners. Expansive learning is manifested primarily as changes in the object of the collective activity. In successful expansive learning, this eventually leads to a qualitative transformation of all components of the activity system. (Engeström & Sannino, 2010, p. 8)

Traditional learning theories see an individual as a separate acting subject and learning as a process in which individuals acquire stable knowledge that can be identified with changes in the subjects' behaviors. In this kind of situation, the teacher knows in advance what students are to learn (Engeström, 2005). Learning cannot be predicted in advance in problem-solving activities with robots because the learning process depends on the students' collective and individual choices during the activities. For example, the teacher cannot predict what type of mathematical tools her students are going to use when solving problems.

In expansive learning, owing to the transformative processes in the activity, the change in the object provides wider learning possibilities. The changes in the object constructed by the learners provide opportunities for them to learn "something that is not yet there" (Engeström & Sannino, 2010, p. 2). According to Engeström (2005, p. 64) "[a]n expansive transformation is accomplished when the object and motive of the activity are re-conceptualized to embrace a radically wider horizon of possibilities than in the previous mode of activity."

During the development of activities, tensions might arise in or between different components in the activity system or between different activity systems. These tensions often change the activity in an innovative manner and create the possibility of expansive transformations. Some participants might question and redirect the activity as a result of contradictions and tensions. That can cause deliberate collective efforts towards change in the activity. A change in the object with several possibilities causes expansive transformations. This kind of collective and transformative process is a part of expansive learning (Engeström, 2005).

Engeström in his later works focused on expansive learning among several activity systems and paid less attention to separate activities. We analyze one activity, the group activity in the classroom, and therefore use primarily Engeström's earlier work.

Research methods

Research context and design

This study was conducted in one primary school in Norway. Norwegian schools are interesting because programming is becoming a part of the mathematics curriculum in Norway, and its school system has a positive attitude towards technology (Utdanningsdirektoratet, 2013, 2018). The school that we chose is of interest for this study because it represents a regular Norwegian medium-sized lower secondary school. The cooperation was natural because one mathematics teacher in that school was about to integrate robots into his teaching in an elective class called "Technology in Practice."

The compulsory Norwegian school consists of a 10-year elementary school. The education is based on the national curriculum in which mathematics has a central role. Mathematics is seen as a part of cultural heritage and the basis of logical thinking. Problem solving is seen as an important component in mathematical competence. In any event, although programming is not yet part of the curriculum in Norway, technology is still strongly present. The use of technology is recommended in most of the mathematical activities (Utdanningsdirektoratet, 2013).

A variety of research strategies were discussed. We wanted to study everyday educational practice, since it is not our aim to analyze or test a best case. Furthermore, we needed a strategy that will enable us to follow the robot activity in great detail in terms of action, interaction, conversation, arguing, tensions and conflicts, interaction with the robots, and use of different bits and pieces of mathematics. To analyze processes and development, we needed to follow the same students over a period of time. We decided to follow one class for one semester, with a combination of observation and video recording of all sessions. Investigating social practices in natural settings is a characteristic of ethnography, understood as in Madden (2017, p. 16): "Ethnography is a qualitative social science practice that seeks to understand human groups (or societies, or cultures, or institutions) by having the researcher in the same social space as the participants in the study."

More specifically, we followed a study design called focused ethnography, which differs from traditional ethnography, for instance, through more time-intensive fieldwork, the role of the researchers, and focused observations with key informants (Skårås, 2018). In classroom studies, which followed the design of focused ethnography, it is natural to use videotaping as a data gathering method because the videotaped data material suits well the study of complex processes of learning and teaching (Skårås, 2018). Our study differs from traditional ethnography also with regard to our role as researchers. The teacher conducting the elective class on robots whom we followed lacked any knowledge of Lego Mindstorm. Consequently, we provided him with a short introduction and discussed issues between class sessions. However, the teacher himself planned the class.

Data collection

More specifically, we conducted focused ethnographic fieldwork by videotaping and writing field notes in order to understand the real activity in the classroom when robots entered the scene.

The teacher followed a plan for introducing robots in an elective class for 31 students aged 12–15. The class allowed for more time and space for explorative and creative robot activity than a regular math class would have. Students at ages 12–15 have knowledge both of technology and mathematics. Although, as researchers, we looked for connections with the mathematics curriculum, we did not want to force it. Our interest was in how the students used the mathematics that they had learned. Thus, students and teachers were free to work innovatively without curriculum pressure. Conversely, the fact that the teacher was a mathematics teacher made the connection with mathematics easier.

The assignments designed by the teacher were open; the students were given the opportunity to create their own designs within the tasks, such as what kind of track they programmed the robot to drive on. The open nature of the task enabled a free environment for activity development. The teacher guided the students' activities when it was possible and when they needed to obtain the collective learning. Most of the tasks concerned driving along a particular kind of track with the robots. Some of the tasks were competitive in nature. Students worked in groups of two to four for practical reasons.

The data gathering took place during eight 75-min sessions by observing one group of three students. The students in the group, "Oscar," "Lucas," and "Jacob," were 12–13 years old. This eight-week period was the time required to see the students' entire development from the introduction of the robots to the smooth use of mathematical tools with the robots. The group selection was based on observations and experiments with videotaping during the first sessions. First, it appears that this particular group of students was one of the groups that seemed to enjoy working with robots. Second, their attitude towards the video camera was natural. However, only the last five sessions were videotaped in full because the three first sessions were concerned mostly with building robots and becoming familiar with them. During our systematic observations, the special focus was on changes in the activities, such as changes in objects and tools.

Data analysis

The analysis was divided into three parts. First, the most relevant and interesting video clips from selected sessions that concerned thinking about the use of mathematical tools were transcribed. For this article, we analyzed two sessions in which mathematical tools were in use. The selection of these sessions was based on our observations and field notes. The transcriptions gave detailed accounts of the conversation but not the actions and interactions of the students and the teacher, their bodily and emotional expressions, and the actions of the robot. Therefore, we supplied the transcriptions and our field notes from observation with detailed field narratives based on watching the video clips.

In the second phase of the analysis, we used the whole activity system triangle in CHAT. The transcribed material and our field narratives and notes were coded with the key concepts from the CHAT triangle, namely, tools, subject, object, rules, community, and division of labor. This was done in order to receive a broader view of the activity development.

In the final step of the analysis, in order for the findings and arguments in the article to be pointed out clearly enough, the analysis limited to the relationship between actors, tools and objects. The deeper analysis focused particularly on the use of mathematical tools and the object development in order to answer the research question. As the aim of this study is to discuss the use of mathematics in robot-based activities, the focus was on tools. Furthermore, as the use of different tools depends on the object of the activity (Engeström, 1987), the focus was also on objects of activities. We conducted the deeper analysis by analyzing the relationships between the codes, particularly the relationship between tools and objects, and by analyzing the changes and developments in the codes and code-relations over time. The role of the teacher was obviously important, but also the students' involvement and preparation, their mediation of the response, division of labor, rules and community. The point of the article is not to identify different causal factors throughout the activity, but to interpret it as mathematics having a changed role in the case by focusing on the dynamics between tools and objects.

We determined the object of the activity by identifying the goal or aim, which all subjects of the activity aimed collectively to reach. The tools of activity were identified with the help of the object of the activity. Subjects of the activity needed certain tools to reach their object (Engeström, 1987). The difference between objects of the activities and tools in use was visible by identifying the focus in the activities. The focus of subjects can only temporarily be on tools (Engeström, 2005). For instance, when students are programming the robot, they may need certain mathematical formula or algorithm in order to get a certain value for their program. When students are using that formula or algorithm, the focus is temporarily on mathematical tool. When students obtained the needed result from their calculation, they used it to reach their object, which was to program the robot. The focus was therefore not on mathematics anymore.

Findings

The data of this study were derived from two different sessions. During these sessions, the students attempted to solve a variety of problems, which were partly designed by the teacher and partly by the students themselves. These sessions are briefly presented and then analyzed in more detail.

During Session 1, Oscar was absent, and Lucas and Jacob had difficulties with collaboration. They showed no enthusiasm in working with the robot. Lucas played with the Lego bricks, and Jacob became frustrated with him. Accidentally and by trial and error, they succeeded in programming the robot to drive along a circle with almost the same starting and ending points. At that moment, the teacher was observing the robot's movements together with the students. On the basis of this observation, the teacher suggested that the students could program the robot to drive along a circle with a radius of 1 m. The students accepted that suggestion and worked with enthusiasm.

In order to program the robot to drive in a circle with a radius of 1 m, the students needed to know how long the robot has to drive and how much it has to turn. The students started solving the problem by determining how much the robot must turn during one wheel rotation and how long the robot must drive using proportions and the circle circumference formula.

The whole turn in EV-3 programming environment is equivalent to the value of 100. With the help of proportion students found out that the value 1 is equal to a 3,6 degrees turn. After that, they found out that the robot has to turn 19.5 degrees during one wheel rotation, because the robot drives 18.5 cm during one wheel rotation and 360:18.5 = 19.5. Furthermore, the students divided 19.5 by 3.6. They used the value 5.5 in their program.

When the students determined the distance that the robot has to drive, they committed an error with the given circle circumference formula, using the radius rather than the diameter and they came up with the answer 3.1415 meters. Thus, the robot drove only half a circle. On that basis, Jacob concluded that they had to double the distance, and the students succeeded with their task.

The students were excited about succeeding in this task, and during Session 2, Lucas and Jacob were willing to apply their learning in a new situation. At the beginning of Session 2, the students were given the new task of driving along a track with the robot, taking hold of a little box, and moving the box along the same track back to the starting point. The students were free to design the track the robot was to drive along by themselves. Lucas and Jacob wanted to have a circle track as a part of the robot's track.

The activity development during these sessions is analyzed in the following section using activity system analysis by focusing on the use of mathematical tools. As the use of tools depends on the object of the activity (e.g., Engeström, 1987), our further analysis concentrated on the object development in the activity.

Based on our analysis, the activity development is divided into four different phases. These phases are discussed and justified in more detail in the following subsections. However, in order to clarify and make it easier to follow our analysis and findings, we present the different phases in the activity development in table 2.

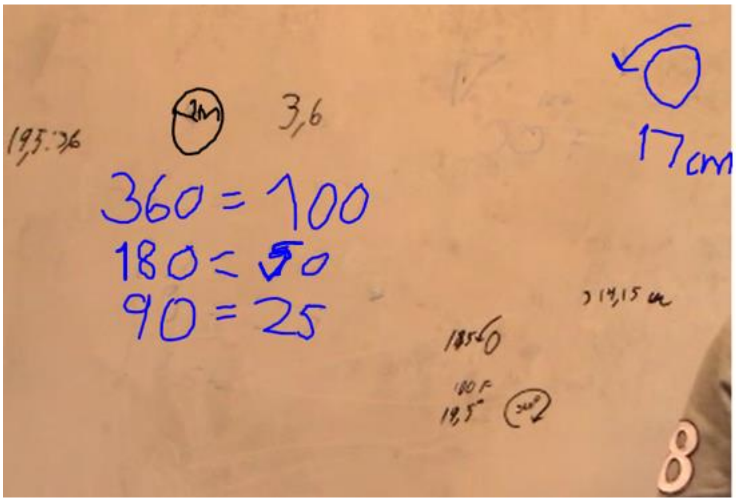
Table 2. The summarization of the components of object of activity and mathematical tools in use during the different phases in the activity development.

	1. The task design	**2. The use of mathematical tools**	**3. Mathematical tool as an object**	**4. Expansion of the object**
Object of the activity	Students started by programming the robot to drive a circle. The teacher mathematized students object by negotiating with students.	The mathematized object, namely to drive the circle with the radius 1 m, enabled the use of mathematical tools.	Because of the error students made with the mathematical tool, mathematics became the object of the activity.	Students wanted to use their learning from last sessions in their new task design. The object of the activity expanded. The new object was to drive a path where a circle track was as a part of the robot's track.
Mathematical tools in use		Students used different types of mathematical tools to reach the object. However, they made an error with the circle perimeter formula.		Mathematical tools were in use again because of the new mathematized object.

Phases 1 and 2: The task design and use of mathematical tools

At the beginning of Session 1, the teacher's suggestion that the students program the robot to drive along a circle with a radius of 1 m motivated the students to collaborate and use mathematical tools. Lucas and Jacob began solving the problem by collaboratively creating their own mathematical tool bank by writing on the whiteboard the mathematical concepts they thought could be useful to them. The students alternated between different roles, with Lucas writing and Jacob suggesting different ideas and vice versa, while they discussed with enthusiasm the kind of mathematical tools they would need to be able to program the robot to drive along a circle with a radius of 1 m. Thus, the teacher's suggestion was the initiator of the students' collective activity, where the driving force, the object of the activity, was to program the robot to drive along a circle with a radius of 1 m. A variety of mathematical tools that the students wrote on the whiteboard mediated the activity. Picture 1 shows a reconstruction of the whiteboard after the students' reasoning.

Picture 1. Reconstruction of what the students wrote on the whiteboard



The teacher's suggestion to program the robot to drive along a circle with a radius of 1 m was a mathematized version of the activity that Lucas had already begun by programming the robot to drive along a circle using trial and error. The detail of the teacher suggesting the use of a radius of 1 m was pivotal in activating the students to use mathematical tools in their problem-solving activity. The teacher mathematized the students object. Here we understand a mathematized object as an object, which needs to be achieved with mathematical tools. If the object had been only to drive in a circle, without more precisely specifying the size of the circle, the students could have solved the problem by trial and error by changing the values randomly in the program Lucas created at the beginning of the session without planning to use mathematical tools. The trial-and-error strategy had also been seen in earlier studies as an obstacle to using mathematics in problem-solving activities with robots (Savard & Freiman, 2016).

In any event, students needed to try different kinds of smaller objects in order to achieve their primary object, to drive along a circle with a radius of 1 m. First, the students used the circle circumference formula as a tool to determine the length of the route that the robot had to drive. However, the students did not realize that they made a mistake with the circle circumference formula, even though they had a short conversation about the value of the circle circumference.

> Lucas wrote on the whiteboard $1 \times 3.1415 = 3.1415$ and stated: *Because it is how many meters it has to drive.*
>
> Jacob was a bit skeptical with this: *Does it have to drive that many?*

Lucas: *Three meters. Yes, because we do have a radius of one and that is why it has to drive three meters, point one four or something like that.*

Because of Jacob’s questioning, the students’ focus was on the mathematical tool during this conversation. That state was only temporary because, after this conversation, the students just took the value of the circle circumference as a tool to use and they did not question its validity any further. Thus, the object remained, and mathematics worked as a tool to mediate the activity, even though the focus was temporarily on the tool.

Second, the students used the ratio of the circle circumference to the robot’s wheel circumference to determine how many rotations the robot wheels had to rotate. They knew from earlier sessions that the robot’s wheel circumference was approximately 17 cm. With the ratio 314 cm:17 cm, they used the calculator to determine that the robot wheels had to rotate approximately 18.5 times. This calculation was the result of common reasoning. Both of the students suggested different kinds of relations to determine the number of rotations required. Through common reasoning, they obtained the correct answer.

Third, the students used proportions to determine how to program the robot to make the turn with a proper angle. With Lego Mindstorm robots, it is possible to program turning on a scale of 1–100. The students began by determining what the scale 1–100 means in relation to the turning angle of the robot. Jacob determined that the value of 100 must mean the entire turn (360°). Students used proportions to determine how many degrees the robot turns with the value one.

After a short discussion, Jacob concluded: *50 is 180 degrees. And then, 25 is 90 degrees.*

The discussion of proportions continued later. Meanwhile, they determined how many degrees the robot had to turn during one wheel rotation.

Lucas: *The robot has to spin 360 degrees, so it will be 360 divided by 18.5.*

Lucas calculated 360:18.5 using the computer and obtained an answer of approximately 19.5. Then, the students continued using proportions to determine what value they had to use to program the robot to turn with the correct angle.

Lucas: *Because, we only have up to 100, we have to divide 360 by 100, which is 3.6. Isn’t it?*

Jacob: *Yes, 3.6.*

> Lucas: *Yes, I hope that is correct. 3.6, ok, so that means that 1 is the same as 3.6 and we have to divide 19.5 by 3.6.*

After a discussion, Lucas used a calculator to obtain the answer 5.5.

> Jacob: *Let's try this out.*

With the help of these different proportions, the students found the correct values to use in their program to make the robot turn in the desired angle. According to our analysis of Jacob's last statement, after the students obtained the required values with the help of mathematical tools, they were ready to test the values in their program, and their focus shifted again from the tools to the object.

In summary, in each of these respects, mathematics was used as a tool for reaching the object, to make a robot drive along a circle with a radius of 1 m. More specifically, the circle circumference formula was used to determine the distance the robot had to drive, and proportions were used to determine how much the robot had to turn. Even though the students' attention was temporarily on the tools, these mathematical tools still remained as tools and not as objects of the activity. After the students obtained the required answers using their mathematical tools, they were willing to use their answers to make the robot drive along a circle with a radius of 1 m, the object of the students' activity. Students simply fed the required values into their program and tested it. The students' focus then was on the testing of the program and on the robot, no longer on the mathematics.

The discussed mathematical tools, namely, circle geometry and proportions, can both be connected with the mathematics curriculum. According to earlier studies, the connections between robot-based activities and curriculum have been unclear (Alimisis, 2013; Benitti & Spolaôr, 2017; Karim, 2015). As the students did not receive any external help, such as information or advice from a teacher, a book, or the Internet regarding mathematical tools, the students used the mathematics that they already knew. However, the use of mathematical tools occurred through collaboration between the students. Both of the students contributed when they were designing the use of mathematical tools or when they were using the mathematical tools. The students alternated between different roles, alternately coming up with different ideas, conducting different calculations using the computer, or writing their ideas and calculations on the whiteboard. The students used the mathematics that they already knew, but their knowledge was strengthened through collaboration, and they were able to apply their knowledge to mediate the activity.

In any event, the use of mathematical tools did not always follow the formal mathematical rules, for example, when the students wrote on the whiteboard 360 = 100, which means that the entire turn of 360° is equal to the value 100 in the program (see Picture 1). The use of free rules in robot-based activities makes the use of mathematics more informal, and thus, students forget the use of formal rules in mathematics. This is in alignment with the argument of Barak and Assal (2018) regarding the challenges of teaching and learning formal mathematics through informal activities with robots. According to Barak and Assal (2018), the informal nature of robot-based activities makes the formal use of mathematics or other science, technology, engineering, and mathematics (STEM) subjects challenging. Furthermore, during free activities with robots, the teacher refrains from interfering in the informal use of STEM subjects.

Phase 3: Object development

The use of mathematical tools depended on the object development. As discussed earlier, the common object, to program the robot to drive along a circle with a radius of 1 m, induced students to design the use of mathematical tools. The further development of the object is discussed in the following.

At some point, when the students were working with the mathematical tools, the teacher realized that the students made an error with the circle circumference formula. The teacher attempted to encourage the students to pay attention to their mistake with the mathematical tool when the students were conducting their reasoning to determine the values required to program the robot.

> The teacher: *I am just wondering, where, how, you got 314.15 centimeters from?*
>
> Jacob looked skeptically at the teacher: *How? What?*
>
> Both of the students looked at the whiteboard, and Lucas gave the answer: *Oh, yes. Because we had to multiply one meter by pi and we had 17 centimeters with one wheel rotation, so we transformed it to centimeters.*
>
> Jacob continued: *So, we ended up with that it has to drive 314 centimeters*.
>
> The students continued working, but the teacher did not give up: *How did you determine to multiply the radius by pi?*

> Lucas looked at the teacher skeptically and then looked at Jacob. He laughed a bit uncomfortably: *I do not know how, I do not remember it now.*

The students continued working without paying any more attention to the teacher's question. They did not want to pay any attention to their mathematical tool in that phase because they were committed to their object and they did not see any point to it. According to our analysis regarding the students' gestures, such as skeptical looks and uncomfortable laughter, the students did not have any idea what the teacher was talking about, and they did not care because they did not want to pay attention to the teacher's question or the mathematics behind the question. Both the mathematics and the teacher were excluded from the object of the students. They just wanted to continue towards their object to drive a circle with a radius of 1 m, the driving force in this phase. Thus, the teacher could not change the students' focus or object with his questions in this phase (e.g., Engeström, 1987).

In any event, after the students had input the needed values, obtained using the mathematical tools, into the program, they tested it, and the robot drove only a half circle. To determine what went wrong, both of the students concentrated and shifted their focus temporarily to the mathematics again. Lucas went through their calculations on the whiteboard while Jacob sat on the computer.

> After some thinking, Jacob suggested: *We try 37.*
>
> Lucas: *Why?*
>
> Jacob: *It is double, because the robot drove only half way.*
>
> Lucas accepted this solution with a smile: *Good plan, we say.*

However, the focus on mathematical tools was only temporary because, after Jacob's suggestion, the students just doubled their answer, used it on their program, and succeeded in their task. Lucas did not even realize why Jacob wanted to double the answer, but his smile showed his satisfaction with Jacob's suggestion. The students just wanted to reach their object, and they were not further interested in the source of the error or the reason why they had to double their answer. They were only interested in reaching their object, to make the robot drive along a circle with a radius of 1 m.

Finally, the students succeeded in reaching their object, their goal, and they were satisfied with the result. Next, there was space for negotiation regarding a new object, a new motive or goal.

After the students succeeded, the teacher continued: *But now you have to determine why*.

Jacob: *We just doubled it.*

The teacher: *Yes, why did you double it?*

Jacob: *Because we saw that it drove only half way*.

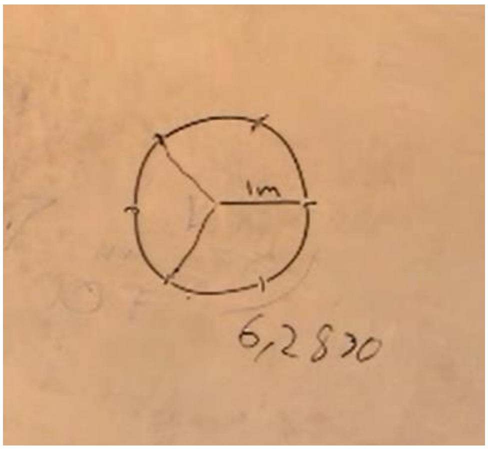
The students were excited about their success, and they were not interested in thinking about it further.

Jacob: *But we managed to do it before the end of this session*.

The teacher: *Yes, that is impressive. But tell me what you calculated*.

After a couple of jokes, Lucas smiled and accepted the teacher's suggestion to explain why doubling the answer worked. The students and the teacher discussed circle geometry on the whiteboard. Picture 2 is the whiteboard view after the discussion.

Picture 2. Whiteboard view after the students' and teacher's discussion of circle geometry



The teacher drew a circle on the whiteboard and pointed to its radius: *So, this is one meter.*

Then, he pointed to one part of the circle circumference: *So, we estimate that this is about one meter, or is it? Is it about one meter?*

Lucas: *1/3 is one meter.*

The teacher pointed to about one third of the circle circumference: *1/3 from here. So, from here to there? Should we call this one meter, then? Can you draw the arc then, one meter? Say we start here. How far away is about as far as this, then?*

> Lucas divided the circle circumference into three different arcs. The teacher pointed to the radius of the circle: *As far as here, about?*
>
> After some hesitation from the students, the teacher pointed to the radius again: *Can you show with your fingers how long you think this is then? How long?*

This discussion continued and Jacob divided the circle circumference into arcs of approximately the length of the radius. Students determined that the circle is approximately 6 m in length.

> The teacher continued: *Yes, how do we find the circumference of a circle then?*
>
> Lucas: *You have to take it twice as much as it is and then…Double radius multiplied by pi.*
>
> This discussion ended with Lucas's statement: *Now we know, why we had to double it.*

Finally, the teacher moved the students deeper into circle geometry, with a new driving force for the conversation being to determine why they had to double their answer. The students explained their calculations to the teacher, with mathematics becoming the object of the students' activity. The focus was not just temporarily on mathematics; mathematics was the drive and direction in the activity. The students concentrated only on mathematics, because they had reached their original object to drive a circle and they did not have to get back to the original object anymore. Mathematics was not just a tool anymore, where the focus is only temporary. The focus was on the mathematics until students reached their new object, to find out why they had to double their answer.

This change in the students' object was a result of the teacher's steadfast negotiation with the students at the proper moment during the activity development, as the collective object can change during the activity development as a result of the manipulations of the activity participants (e.g., Kuutti, 1996).

The object change was an interesting turning point in the students' learning process. First, without the object change, the students would have been satisfied with reaching their original object. Thus, the students would not have discovered the source of the error, which has been seen in earlier studies as an obstacle to acquiring greater understanding in mathematics (Savard & Freiman, 2016).

Second, before the object change, mathematics was used as a tool, and the students' attention was not on it. When mathematics was used as a tool, students used their mathematical knowledge in action informally. In any event, when mathematics became the object of the activity, the students paid attention to it and obtained new knowledge with it, giving the teacher an opportunity to teach formal mathematics. That mathematics became the object of the activity does not mean that it displaced technology and the robot. On the contrary, the robot and mathematics merged into a hybrid and expanded object.

Third, the object change was dependent on the role of the teacher during activity development. When the teacher realized that the students erred with the circle circumference formula, he could have just corrected the mistake by mentioning it to enable the students to double their answer. That advice would have stopped the development towards object change, and mathematics would have remained just some informal tool in the activity. However, the teacher did not do that but decided to follow the students work and, when possible, to ask relevant questions and negotiate with the students without providing any ready tools to use. The object change was not externally provided but was a result of the process of student activity. The teacher was a mediator in the process.

Phase 4: Expansion of the object

During Session 2, Oscar was again present, with Lucas and Jacob satisfied with their success last time and willing to apply their learning in a new situation. They wanted the robot to drive along a circle of a different size as a part of their new task.

> Lucas: *Did you delete the program, which made it drive a circle with a radius of one meter? It was a program with lots of mathematics in it.*
>
> Jacob smiled: *A lot of mathematics in it*.
>
> Lucas talked by emphasizing the word mathematics and enthusiastically waved his hands: *I have an idea. Now, we are going to do this with lots of mathematics, do you understand? ... Yes, we are going to make that big circle and we are going to use mathematics*.
>
> Lucas started to measure the diameter of the circle, which he had built with Lego bricks with the idea of programming a robot to drive around this circle as part of its pathway on the track that the students were planning to create. Jacob helped him: *How long a diameter does it have?*

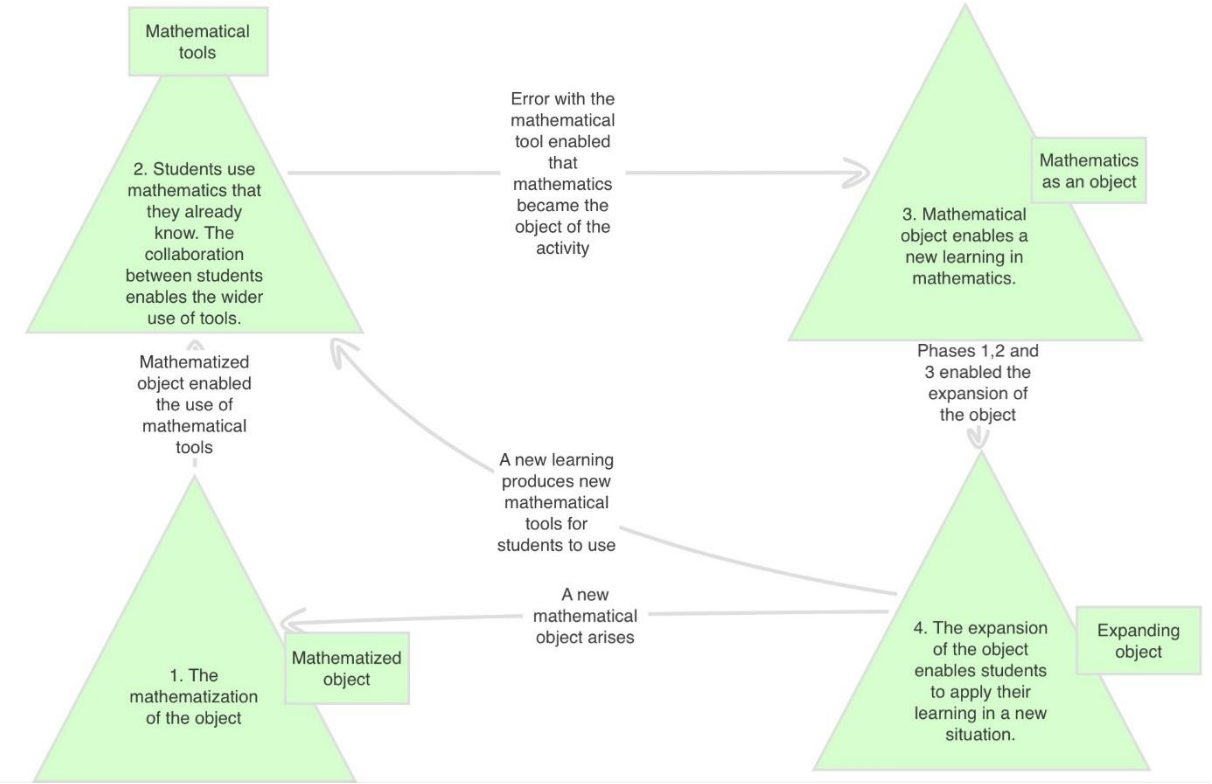
Lucas conducted the required calculations on the whiteboard for a new circle in the new situation. Mathematics was the driving force in this situation. The word mathematics had a positive tone in the conversation; Lucas was excited about mathematics. Lucas' excitement manifested in the way that he talked about mathematics and waved his hands enthusiastically. The word mathematics made Jacob smile, and mathematics was something to strive for. Important for the students was that the new object in a new task had mathematics that they learned last time in it. Because of the students' learning process and their satisfaction with their success from the last session, the students were willing and able to apply their learning in a new, wider situation. The mathematical object expanded (e.g., Engeström, 1987).

In summary, the use of mathematics and the expansion of the object consisted in specific turning points in the activity development (Figure 2). In the first phase (figure 2 and table 2), the students worked towards their object, which was mathematized. The students' original object was to drive in a circle, with the teacher suggesting the size of the circle. That is, the teacher refined the students' original object with mathematical details.

In the second phase (figure 2 and table 2), the mathematized object enabled the use of mathematical tools. At this point, the students used their mathematical knowledge to achieve their mathematized object. The students used the mathematics that they already knew by applying their knowledge in action, which was possible through collaboration. Thus, the use of mathematics was in relation with the students' collaboration. The collaboration between students developed the activity towards their common object.

In the third phase (figure 2 and table 2), the mathematical tool became the object of the activity after the students had reached their original object. Finally (phase 4 in figure 2 and table 2), the object was expanded when the students used their learning in a new wider context. The students created a new activity with a new mathematized object, which enabled the use of their new learning as a tool to mediate the new activity.

Fig. 2 Different turning points in the activity development towards the expansion of the object (Inspiration Maps).



None of these turning points alone would have enabled this kind of development. Thus, these different steps are strongly intertwined with each other. Furthermore, all of these steps stem from the students' first object, to drive a circle with a robot, which was the driving force during the entire process. These steps would not have been realized without the original object with the robot.

Conclusions and discussion

Our review of existing studies showed that the educational benefits of robotics in mathematics education are unclear, at least in part because they occur in complex environments involving digital tools, mathematical concepts and alternative pedagogies. In our research, we addressed the question of students' learning through analysis of the object development, not as changes in subjective knowledge, which quantitative studies on students' learning concentrate on.

Qualitative discussions regarding students' learning processes with robots indicated that the trial-and-error strategy for solving the problems functioned as an obstacle to finding the source of the error with the mathematical context (Savard & Freiman, 2016). By contrast, we introduced one case in which students avoided the trial-and-error strategy and used mathematical tools in robot-based activities. In the perspective of CHAT, the trial-and-error strategy differs from the activities of this study regarding the mathematical tools in use. In a trial-and-error strategy, students select the needed values randomly and mathematical tools are not used systematically. In this study, students did not select the values they needed in their programming randomly; they used mathematical tools systematically instead.

Furthermore, the activity developed towards expansion of the object through the error students made with the mathematical formula. Because the trial-and-error strategy was avoided, the students could detect the source of the error (cf. Savard & Freiman, 2016) and were enabled to create an expanded and hybrid object in which mathematics was merged with technology.

Barak and Assal (2018) argued that, even if problem-solving activities with robots provide rich learning experiences in mathematics, robotics cannot substitute for systematic mathematics teaching. We argue that, even if systematic formal mathematics teaching is not possible with robotics, at least in a traditional teacher-led manner, the teaching of formal mathematics is still possible as found also in Bartolini Bussi and Baccaglini-Frank (2015) with younger students. The younger students in Bartolini Bussi and Baccaglini-Frank (2015) deepened their understanding about rectangles in a practical context with robots. In this study, through the mistake that the students made with the circle circumference formula, the teacher took advantage of an opportunity for a thorough teaching session in circle geometry. A clear and practical connection with robots made the formal teaching session special and rich. Students learned how to use circle geometry in practice, their understanding about formal circle geometry deepened. Our finding related to the connections between formal mathematics and robot-based activities strengthens the idea of Bartolini Bussi and Baccaglini-Frank (2015) that activities with robots have the potential to open also other formal mathematical meanings for the students.

In any event, the opportunity for a rich learning session was not self-evident. The activity development described above was the result of particular incidents that are not directly generalizable. The students' learning could not have been predicted beforehand as their learning depended on the above-mentioned turning points in the activity development, which depended on choices and decisions that the students made. However, our point is that formal mathematics teaching is still possible with robots. The teacher cannot make a teaching plan that is as clear and as detailed as in traditional mathematics education, but curriculum connections can still be made in a more formal manner through robot-based activities (cf Alimisis, 2013; Benitti & Spolaôr, 2017; Karim et al., 2015).

We argue further that robot activity as analyzed in this case opens the possibility for curricular mathematics to be an integral part of the object of an activity in school. Mathematics is transformed from a means of assisting the joy, energy, and motivation of succeeding with the robot activity to a part of the motivational object itself. Thus, the activities with robots have the potential in mathematics education by providing a motivational environment for mathematics learning.

The limitation of this study is that the teacher of this study did not have a relevant programming background, and thus, the programming task assignments were not as advanced regarding programming. The activity development could have been even stronger with more advance programming tasks, developed by the teacher. This study has concentrated on the activity development in the beginning of robot integration. In further activity development, there is also a need to focus on the programming skills development, for diverse development of robot-based activities in classrooms. Thus, teachers' programming skills are what needs to be considered in mathematics teacher education and in teachers' further education. Anyhow, the role of the teacher in students learning processes with robots in the situation where the teacher does not have any programming background is discussed more detailed in our earlier article (Forsström, 2019).

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Full length article

Role of teachers in students' mathematics learning processes based on robotics integration

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1. Introduction

Many countries, such as the Nordic countries Finland and Sweden, integrate programming into their mathematics curriculum. Norway is in the planning phase of a similar curriculum redesign.

The educational potential of programming in mathematics education is still unclear. In Nordic countries, the rationale for introducing programming into the mathematics curriculum is to foster students' logical thinking, problem-solving skills, and motivation to learn mathematics (Bocconi, Chioccariello, & Earp, 2018). According to the report produced by European Schoolnet, “a network of 31 European Ministries of Education,” there is a need for studies to be conducted on ways in which teachers can effectively integrate programming into their teaching (Balanskat & Engelhardt, 2015; Bocconi et al., 2018). On the basis of the literature review about the educational potential of programming in mathematics education, programming integration has the potential to foster students' learning in mathematics and motivation to study mathematics (Forsström & Kaufmann, 2018). However, Forsström and Kaufmann (2018) argued that the results cannot be generalized. In addition, there is a need for studies that discuss the effect of “the role of the teacher” and “collaboration among students” in students' learning processes regarding mathematics. Collaboration among students and the changed role of the teacher are usual in programming activities. Previous studies discussed that the teacher acts more like a guide, supporter, and conflict solver in students' collective learning processes than a traditional lecturer. Moreover, the effect of the roles of the teacher in students' learning is still unclear. This study contributes to the discussion regarding the roles of the teacher in students' learning processes in mathematics based on programming integration.

In general, programming tools fall within the general area of digital technology, which, according to previous studies, can influence mathematics education positively. The curricular integration of digital technologies into mathematics can provide innovative learning environments in mathematics classrooms, such as a creative task design and a new kind of division of labor between students and teachers in the classroom. It has the ability to change a traditional, teacher-led mathematics classroom to be more student-centered (Bray & Tangney, 2017; Olive et al., 2010). Moreover, the way in which technology is used in the classroom completely depends on the choices made by the teacher (McCulloch, Hollebrands, Lee, Harrison, & Mutlu, 2018).

The propensity of any of these potential benefits of programming integration to be achieved depends on the teacher's computing background. When programming is integrated into the mathematics curriculum, the teaching of programming becomes the responsibility of the mathematics teacher.

One way to make programming integration easier for teachers who do not possess any programming background is to use visual programming environments, wherein programming is made possible by the application of different graphical blocks. In visual programming environments, different figures represent different programming structures, such as loops and if-statements. One can then program by changing the values of variables in the figures (Bocconi et al., 2018). The EV3-programming environment for Lego Mindstorms robots is one example of a visual programming environment that is currently used in schools. Lego Mindstorm robots are the most studied educational robots (Benitti & Spolaôr, 2017).

In the educational application, there are various types of robots and toolkits (Karim, Lemaignan, & Mondada, 2015). According to the literature on educational robotics, it is still unclear as to how teachers can fruitfully integrate robotics into curriculum activities

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(Benitti & Spolaôr, 2017; Karim et al., 2015; Mubin, Stevens, Shahid, Mahmud, & Dong, 2013). The studies focusing particularly on robotics in mathematics education brought out the need for discussion of how the role played by the teacher in robot-based activities influences students' learning of mathematics (Lindh & Holgersson, 2007; Savard & Freiman, 2016). Earlier studies discuss how robot integration affects students' performance in mathematics and their motivation to learn mathematics (Ardito, Mosley, & Scollins, 2014; Barak & Assal, 2018; La Paglia, la cascia, Francomano, & La Barbera, 2017; Leonard et al., 2016; Lindh & Holgersson, 2007). Changing the role of the teacher from that played in a traditional classroom is mentioned in some of the studies (Lindh & Holgersson, 2007); however, such changes are not widely discussed as components of students' learning processes with robots.

To integrate programming into mathematics education, Lego Mindstorm robots have the means to accomplish such smooth programming integration. This study focuses on the integration of Lego Mindstorm robots into the learning processes of mathematics. In addition, as the role of the teacher in successful robot integration into mathematics education is unclear, the study aims to specifically discuss what roles teachers can play in students' learning processes regarding mathematics in robot-based activities by answering the following question:

How does the role of the teacher in robot-based activities influence students' learning processes in mathematics?

This question is investigated by comparing two different sessions from data gathered in one secondary school in Norway in which a mathematics teacher without any previous knowledge of programming introduced Lego Mindstorms robots to his students. We will concentrate on what changes occur to the everyday practice of the classroom when programming and robots are introduced. When the teacher does not have any previous knowledge of programming, such an introduction is not always an ideal one. The focus of this study is not what a teacher should do in the classroom but what the teacher actually does and what kind of choices they make in the process of introducing learning with robots. We will concentrate on the role played by the teacher in the collective learning processes of one group of three students, aged 12–13. During the first session, the students were unable to complete the assigned task with the robots. During the second session, they were more successful. Furthermore, during the second session, their use of mathematical tools increased.

We analyze the role of the teacher in the collective learning processes of students using Engeström's (1987) activity system analysis in cultural–historical activity theory (CHAT). CHAT enables the analysis of the effect of various components, such as the role of the teacher in collective learning processes. CHAT considers teaching to be one part of students' learning processes and enables researchers to analyze learning and the teacher's role in collective learning processes where the teacher is a participant (Engeström & Sannino, 2012). In this model, knowledge is distributed among participants and tools. Learning is thus viewed and analyzed as a development of the collective knowledge of a group rather than as knowledge transferred from the teacher to the students (Engeström, 2005). In the sessions analyzed as part of this research, teaching and learning were affected through innovative, collective group processes, as well as through interactions between students and the teacher. Our analysis concentrates on student–teacher relationships as measured through interactions and negotiations between the teacher and the students during the open learning processes with robots.

The rest of the paper is organized as follows. First, existing literature on the role of the teacher in robot-based activities is discussed. A discussion of the theoretical framework of this study comes next. Then, we present our research strategy and methods, detailing how the two different sessions are analyzed and compared using concepts from activity system analysis. In the final section, we review the findings and discuss ways in which the findings can be generalized.

2. Literature review

2.1. Robots in mathematics education

The various studies on the application of robotics in mathematics education mentioned the changed role of the teacher in classroom activities (Barak & Assal, 2018; Lindh & Holgersson, 2007). The studies reported that even if the role of the teacher differs from the teacher's traditional role as a lecturer, the teacher still has an important role to play. Students need the teacher as a guide and support while learning the technology. At times, students also need help to solve technical problems before they can continue programming and working with robots. Teachers with previous knowledge of technology, physics, or natural science are able to link their knowledge to Lego technology (Lindh & Holgersson, 2007). Barak and Assal (2018) reported that students sought the teacher's advice in the beginning. Once they got control over the robot, collaboration between students took a more central role than that of the teacher. Students received feedback from each other, rather than the teacher, when successfully completing a task. However, the teacher was still needed to help in ways other than answering questions. For example, according to Lindh and Holgersson (2007), the teacher is needed to help in resolving conflicts among students and to effectively improve their collaborations.

Moreover, even if the changed role of the teacher is mentioned in previous studies, the deeper discussion on ways in which the role of the teacher influences students' learning processes is missing in mathematics (Forsström & Kaufmann, 2018; Savard & Freiman, 2016). Thus, in this study, we extend this literature review to include programming and more generally, integrate technology into mathematics education.

2.2. Technology in mathematics education

The idea of integrating programming into the mathematics curriculum was introduced in 1980. Papert (1980) suggested that programming should be a part of the curriculum. The author remarked that programming integration can provide a different kind of learning environment in a mathematics classroom, where also the role of the teacher differs from that of a traditional lecturer. In fact, he stated that "the role of the teacher is to create the conditions for invention rather than provide readymade knowledge" (Papert, 1996). In addition, he stated that "I am convinced that the best learning takes place when the learner takes charge…" (Papert, 1993, p. 25). Different studies have been conducted on programming and technology integration into mathematics education on the basis of Papert's suggestions. From this perspective, we discuss how the role of the teacher can help in mathematics education.

According to the systematic literature review by Bray and Tangney (2017), technology integration in general and programming integration in particular have the potential to change classroom culture in mathematics education. Previous studies dealing with technology integration into mathematics education discussed learning environments that were self-directed (Bray & Tangney, 2017; Martinovic, Freiman, & Karadag, 2013; Olive et al., 2010) or student-centered (Bray & Tangney, 2017; Olive et al., 2010), and collaborative (Martinovic et al., 2013). Some studies refer to various challenges in the traditional, teacher-led mathematics classroom (Bray & Tangney, 2017; Martinovic et al., 2013; Olive et al., 2010).

One significant issue in mathematics education is that in many schools, mathematics is still presented as an isolated, formal, and abstract subject, in which the absolute authority is the teacher, whose role is to be the arbiter of knowledge (Bray & Tangney, 2017). The problem with this kind of learning environment is that it hinders students' abilities to make out-of-school connections with what they are learning (Olive et al., 2010). Digital technologies have the potential to address this issue by providing students with more practical connections to the course material, as well as by offering a pedagogical approach that puts the student at the center of their learning environment. In a student-centered learning environment, students are given the opportunity to design their own technologies and to lead tasks with out-of-school connections. Furthermore, the teacher is able to act more like a guide and less like an inveterate lecturer who gives students contrived tasks to solve (Bray & Tangney, 2017; Martinovic et al., 2013; Olive et al., 2010). Although a great deal of innovative research has been conducted in this area, when it comes to everyday classroom practices, technology still tends to be used as a convenient tool for solving traditional problems rather than as a central object of inquiry in a student-centered learning environment (Bray & Tangney, 2017; McCulloch et al., 2018). Earlier studies suggested that our general unwillingness to integrate technology into the mathematics classroom might well be due to teachers who are themselves unfamiliar and uncomfortable with technology in the context of pedagogy (McCulloch et al., 2018). McCulloch et al. (2018) argued that teachers primarily use technology to support pedagogical models and goals with which they are already familiar. Teachers who integrate more "non-mathematics-specific" technology in their pedagogy tend to possess a broader understanding of technology integration (McCulloch et al., 2018).

The role of the teacher in integrating technology into education has been discussed in earlier studies with the help of the technological, pedagogical, and content knowledge (TPACK) model (Ruthven, 2014). The TPACK model enables researchers to analyze how the teacher's technological, pedagogical, and content knowledge influences technology integration (Koehler & Mishra, 2009). Student-centered integration of technology ultimately depends on the teacher's technological knowledge (Guerrero, 2010). Furthermore, reality differs from the ideal situations that research seeks to present. Researchers can dream up new ideas and new models; they can provide devices and assistance to teachers. However, when teachers are left to do their jobs, they themselves will decide what the changes should be (Bray & Tangney, 2017). A further consideration is that teachers may be afraid of losing control of the classroom (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Olive et al., 2010). In the traditional mathematics classroom, the teacher has the control and the power to make decisions. They can decide how technology is used. If students are given more control and responsibility in the classroom through technological integration, they will be forced to engage in a new set of difficulties: pedagogical and otherwise. (Olive et al., 2010)

In summary, technology has the potential to change classroom practices in mathematics education. How the classroom changes depends on the manner in which teachers integrate technology into their classrooms (Bray & Tangney, 2017; Olive et al., 2010), which itself depends on the teacher's technological knowledge (e.g., Guerrero, 2010). It remains unclear as to how a teacher who does not have previous knowledge about programming or robots manages to integrate robots into their classroom successfully.

3. Theoretical framework

Teachers with strong technological knowledge are able to integrate technology into their teaching in a student-centered way (Guerrero, 2010). In such cases, the integration of technology has the potential to change the classroom culture in mathematics education. It is very likely, however, that a teacher integrating programming in their classroom lacks the technological knowledge of someone with a programming background (Bocconi et al., 2018). As we focus on the choices of a teacher without technical expertise, we apply a different theoretical perspective than the TPACK model. Rather than focusing on the teacher's knowledge or intentions toward integration, we aim to understand what actually happens in the classroom by using a wider relational perspective. We discuss the influence of the role of the teacher on students' learning processes by addressing all the components of collaboration among students, student-centered classroom environments, and teachers' technological knowledge. This is possible with CHAT.

Engeström's (1987) activity system (Fig. 1) is the prime unit of analysis in CHAT. Engeström's (1987) activity system analysis considers the role of the teacher in relation to the collective learning processes of the students. The activity system analysis in CHAT considers tool-mediated activities in light of all the components of the activity system (Fig. 1). The seven components considered in relation to one another are subject, object, tool, rules, community, division of labor, and outcome (Table 1).

Activity is always led by an object, which motivates and determines it. Subjects work collectively through the division of labor toward their collective object (Engeström, 1987). The collective object of an activity can be collaboratively constructed by subjects and can be the synthesis of individual perspectives (Engeström, 2008; Holland & Reeves, 1996). The object can be shaped and changed in the course of the development of the activity by the subjects (Engeström, 1987, 2008).

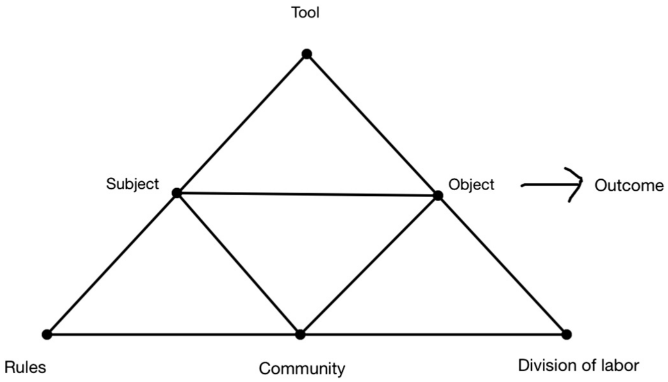

Fig. 1. The activity system model (Engeström, 1987, p.78).

Table 1
Definitions of the components of the activity system analysis.

Component	Definition/meaning	Examples from this study
Subject	The individual or group of people who are engaging in the activity (Yamagata-Lynch, 2010)	The students and the teacher
Object	The driving force of the activity (motive and goal) (Engeström, 1987)	Fulfill a task with the robot.
Tool	The instrument that mediates the activity (Engeström, 1987)	The robot, computer, and mathematical tools
Rules	The regulations that are relevant to the activity (Yamagata-Lynch, 2010)	Task assignment, rules of the mathematics classroom
Community	The social group to which the subject belongs during the activity (Yamagata-Lynch, 2010)	The whole class of students and the teacher (or teachers)
Division of labor	How the tasks are shared during the activity (Yamagata-Lynch, 2010)	Collaboration between students, the mediation of the teacher
Outcome	The result of the activity (Yamagata-Lynch, 2010)	The robot drives a track as it is programmed.

Activities are mediated by tools as subjects are connected to objects through the use of the tools that they possess. There cannot be any activity without tools. Moreover, the tools are not the main objective or the goal of an activity even if the focus is temporarily on the tools (Engeström, 2005). The tools required for robot-based activities can be working tools (Engeström, 2005), such as computers and robots, or nonmaterial tools (Engeström, 2005), such as knowledge of a programming language-specific mathematics.

Engeström's (1987) activity system model is well suited to analyzing the role of the teacher on the basis of activities transforming over time, and the history of the participating subjects should be a part of the analysis of programming and robot integration because CHAT is used for collaborative activities. Each subject has its own history, which shapes the activity through the division of labor. Thus, the role of the teacher in collective learning processes can be analyzed using the relationship between the role of the teacher as a part of the division of labor and other components in the activity system, such as various tools and the object of the activity. Programming tasks can be solved in various ways because collective activities are not predictable (Engeström, 2005). Consequently, even if the teacher designs the tasks, they may not know beforehand the type of problem-solving activities that will be set in motion. Therefore, this may change the division of labor in the classroom.

In this regard, Engeström and Sannino (2012, p. 46) stated the following:

> But the very assumption of complete instructional control over learning is a fallacy. In practice, such control is not possible to reach. Learners will always proceed differently from what the instructor, researcher or interventionist had planned and tried to implement or impose.

Thus, learning processes using robots cannot be analyzed with standard learning theories wherein students are considered to acquire stable individual skills and knowledge that can be identified by the teacher beforehand. As aforementioned, learning in CHAT is a collective process. The subject of an activity collaboratively manipulates the object of the activity throughout activity development (Engeström, 2005).

To understand the role of the teacher in students' learning processes in detail, we discuss the relationships among “the role of the teacher,” “object of students activities,” “tools in use,” and “collaboration among students” in student activity system models. These relationships are the main focus of the analysis in this study.

3.1. The role of the teacher-object of the activity

As aforementioned in the introduction, technology integration has the ability to change classroom culture from being teacher-led to becoming more student-centered, depending on the role of the teacher. Engeström (2008) illustrated the traditional classroom model using activity theory (Fig. 2). In that model, the teacher and students have their own activity systems, where the teacher sets

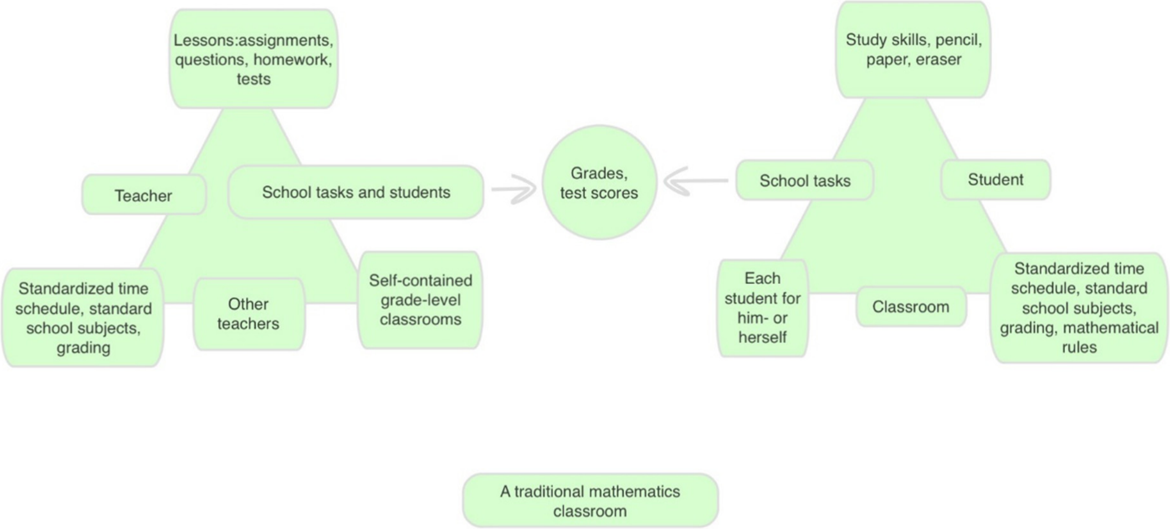

Fig. 2. Engeström's illustration regarding a traditional classroom model reconstructed from Engeström (2008, p. 89).

up tasks for the students to solve, i.e., the object of the students' activity is given (Engeström, 2008; Martinovic et al., 2013). In a more student-centered classroom, students have the opportunity to create their own tasks to be solved, i.e., students have their own objects. However, previous studies have shown that there may be a problem in integrating programming, particularly if the teacher does not have any programming background to enable the students to obtain their own objects. To understand this kind of activity in the classroom, we address the problem of the relationship between the role of the teacher and the object of the activity in students' activity systems. Thus, the relationship between the role of the teacher and the object of students' activity system is one main focus of analysis in this study.

3.2. The role of the teacher- tools

According to the previous studies that applied the TPACK model, technology integration into mathematics education depends on the teacher's technological knowledge (Guerrero, 2010). In CHAT, teacher and student technological knowledge, as well as teacher pedagogical knowledge, can be discussed by utilizing the concept of tools. Activities transform over time, and the history of the participating subjects should be a part of the analysis. Activity development depends on the history of the different tools that mediate and shape the activity (Engeström, 2005). The students and the teacher can have different histories regarding programming tools. If programming is integrated into mathematics education and the mathematics teacher does not have any previous knowledge of it, some of the students may have more control over the knowledge tools than the teacher (Olive et al., 2010). The influence of the teacher's role in student activity systems can be discussed in such a situation by addressing the relationship between the teacher's role and the tools in use, which is our second unit of analysis in this study.

3.3. The role of the teacher-collaboration among students

Previous research proved that technology, programming, and robot integration support collaborative learning in mathematics classrooms (Forsström & Kaufmann, 2018; Bray & Tangney, 2017; Martinovic et al., 2013). CHAT can be used for collaborative activities. Each subject has its own history, which can shape the activity through the division of labor. The subjects participating in the activity can always have multiple perspectives, opinions, traditions, and interests, which can cause tension (Engeström, 2005). Previous studies proved that the teacher needed robot integration as a conflict solver (Lindh & Holgersson, 2007). However, the influence of the teacher as a conflict solver on students' learning is not widely discussed. Activity system analysis in CHAT enables us to discuss the influence of the teacher's role as a conflict solver on students' learning processes in mathematics, through the relationship between the teacher's role and collaboration among the students. This constitutes our third unit of analysis in this study.

In the following chapter, we introduce the design applied to our study. We use CHAT to analyze the teacher's role in robot-based activities and how it can influence the students' learning processes in mathematics.

4. Research methods

4.1. Research context and design

We explore data gathered in one secondary school in Norway to better understand the role of the teacher in student learning when technology is being implemented in the classroom.

Norway is planning to integrate programming in the mathematics curriculum, and technology is already an important part of the curriculum in Norway, which makes a Norwegian school an interesting target for this study. Mathematics plays a central role in the national curriculum in the 10-year compulsory elementary school in Norway, and logical thinking, problem-solving, and the use of technology are the central focus of the mathematics curriculum in Norway. It is recommended that teachers use technology in most mathematical activities (Kunnskapsdepartementet, 2007; Utdanningsdirektoratet, 2013).

Although the design of this study is not completely that of an intervention study, it has features of an intervention study in terms of our role as researchers. We played an active role in the beginning when we introduced Lego Mindstorms robots to the teacher shortly before the data was collected through the basic programming figures. After the introduction, he gained basic programming skills with robots by himself. The teacher planned and conducted an introduction to robots. We had a more passive role during the data collection sessions; however, we negotiated with the teachers in between the sessions, which reactivated our role.

4.2. Data collection

The videotaped data with ethnographic features were gathered to understand everyday activity in the classroom in which robots were integrated. The mathematics teacher who participated in this study did not have any previous knowledge of programming or robots. This made it possible for us to get a natural picture of the situation of programming integration. The data were collected as part of an elective subject, "technology in practice." There were 31 students aged 12–15 in the classroom.

We started by introducing Lego Mindstorms robots to the teacher shortly before the data were collected through the basic programming figures. After the introduction, he gained basic programming skills with robots by himself. The teacher planned and conducted an introduction to robots, first introducing the basic programming figures to the students so that they could steer the robot motors. Other programming skills were self-taught as needed. The students worked in groups of 2–4 that were assigned by the teacher. The task that was most frequently selected for groups was to drive a certain path with the robot. Students were also able to plan the path that the robots would drive by themselves. The teacher, with his peers, guided students when needed.

We observed the activities in the classroom during eight 75-min sessions; the last five of the sessions were videotaped. By observing the first eight sessions incorporating robots, we got a sense of the students' first learning experience with robots. During the sessions, the teacher's involvement with students' learning processes varied.

The focus in our observation was on one group of three students, "Oscar," "Lucas," and "Jacob," aged 12–13. Oscar, Lucas, and Jacob worked enthusiastically with the robots, and they were natural in front of the video camera. Observing only one group of students made it possible to gain a detailed understanding of the collective learning processes of that one group.

Permission for gathering the sensitive videotaped data material was obtained from the Norwegian Centre for Research Data. Permission for the videotaping was also granted by the teacher and the parents of the students. The data has been processed confidentially, and the participants are anonymous. During the sessions, the author concentrated on videotaping and writing field notes. The role of the author in the classroom was that of a moderate participant, which is between an active participant and a totally passive one (Spradley, 1980). The author briefly introduced the robots to the teacher but did not participate in teaching or guiding students, instead concentrating on observing the activity development and the effects of different components such as the role of the teacher, collaboration between students, and different tools in use. Furthermore, the author observed students' and the teacher's gestures and expressions to understand interactions between students and the teacher.

4.3. The data analysis

The first step in data analysis was to select for transcription the most interesting part of the most interesting sessions, with respect to the role of the teacher. Two different sequential sessions were the most interesting for this article because the teacher had a different role during these sessions.

To get a broader picture of the impact of the role of the teacher on students' collective learning processes, we used Engeström's (1987) activity system analysis, which made it possible for us to analyze the influence of the role of the teacher on the different components in students' learning processes. We also compared the two different sessions regarding the role of the teacher, and on the basis of this comparison, we analyzed certain relationships between the role of the teacher and other components in the activity systems in which the role of the teacher had an influence on both of the sessions, which could be seen in the collaboration between students, the tools that the students were using, and the objects of students' activities.

5. Findings

In this section, we briefly present Sessions 1 and 2, and then, we provide a comparative analysis of the roles played by the teacher during these sessions, using activity system analysis.

During Session 1, the task assigned was to program the robot to drive a given path as quickly as possible. Students Jacob, Lucas, and Oscar got the idea to use touch sensors to control the robot in such a way that the robot turned left when the touch sensor connected to the left-hand side of the robot was pressed down. The right-hand side would work similarly. When no sensors were pressed, the robot would drive straight forward. This idea was generalizable and innovative; they were not able to implement it because they lacked the necessary programming skills. The teacher was not able to help them either.

During the other session, 1 week after Session 1, the assigned task was still the same as that in the previous time because none of

Table 2
Comparison between Sessions 1 and 2.

	Session 1	Session 2
Object	The teacher was not present when the students negotiated the object.	The teacher negotiated the object together with the students.
Mathematical tools	Students did not use mathematical tools.	Mathematical tools were in use.
Programming tools	Students had problems with programming. They could not solve these problems.	Students did not have any problems with programming.
Collaboration	The students had difficulties with collaboration.	The students had difficulties with collaboration; with the help of the teacher, the difficulties were resolved.
Outcome	Students did not complete the task.	Students completed the task.

the groups had yet succeeded. Students had the option of designing a path on which they programmed the robot to drive. During Session 2, Oscar was absent, and Jacob and Lucas had difficulties in getting started and collaborating. They had to wait for a long time before the teacher came to help them; however, when he did, he negotiated a new task assignment with the students. The new task was to program the robot to drive a circle of 1-m radius. This negotiation with the teacher helped the students to successfully collaborate again, and they were able to solve the problem using mathematical tools and collaboration.

The roles of the teacher during these two sessions were different in many ways, and so were the sessions themselves. These differences are listed in Table 2.

Using the relationship in the activity system analysis, we discuss the differences between Sessions 1 and 2. For this analysis, the role played by the teacher is considered to be a part of the division of labor. In the following sections, we use Table 2 as the basis for discussing the relationship between the teacher's role and the object of the activity, between the teacher's role and the tools, and between the teacher's role and collaboration between students.

5.1. The role of the teacher - object of the activity

During Session 1, the teacher was not present when the students negotiated their object:

Citation 1 from our field notes (Session 1)

Students found some sensors and became curious about their purpose. After the students learned the purpose of the touch sensor, Oscar got an idea about using it to control the robot. He described how the touch sensor could be used. Jacob got excited about Oscar's idea.

Lucas was not excited about Oscar's suggestion: *We are not going to do that.*

The students connected two touch sensors to the robot with long cables so that the sensor could be pressed while the robot was driving.

At this point, Lucas also got excited: *Let's make a program.*

Oscar started programming while he negotiated with Jacob and Lucas. Thus, the students found a common object to work on without any input from or negotiation with the teacher.

Citation 2 from our field notes (Session 1)

The teacher came to see the students after they had tried to solve their problem for some time.

Teacher: *Do you know what to do?*
Oscar: *We are just testing things out…*
Jacob laughed: *We are trying to make a controller.*
The teacher was a bit confused: *Okay.*
Oscar: *We are having problems in getting this to work somehow. We are having problems in getting these things to work somehow. I mean the buttons…*
The teacher: *What did you say?*

The teacher pointed to the computer screen and took a quick look at the computer, smiled, and left. He did not answer the students' question and did not return to the students during that session. The students did not mind that the teacher left; they just continued working.

Analysis based on Citations 1 and 2

The teacher did not understand the students' object. This is evident because he did not comment on the students' idea in any detail; he just left and did not return. One reason for this is that he was not present during object negotiation. The other reason is that the teacher did not have the tools required to help the students. We discuss more about the required tools in the tools section.

As opposed to Session 1, during Session 2, the students' object was a result of the negotiation with the teacher:

Citation 3 from our field notes (Session 2)

The teacher came to see the students 30 min after the beginning of Session 2. He noticed that one student was absent:

> Teacher: *Are you without Oscar today?*
> Lucas: *Yes.*
> The teacher remembered the students' project from the last time: *Was it he who planned the controller, or?*
> Lucas: *We gave up on the controller.*
> The teacher laughed relieved: *Okay, what are you doing now instead?*
> The students gave almost the same answer at the same time.
> Jacob: *I don't know what he is doing.*
> Lucas connected the robot to the computer and smiled: *We made it (the robot) turn.*
> Jacob continued, a bit frustrated: *I just continued with this thing here (programming the robot), which he started. He was sitting here like this, more than half the time he just built it (Lego bricks). And now I do not know what he is doing with it (the robot).*
> Lucas gave an answer while he started the robot on the floor, and he pointed out a route on the floor with his hands: *Now I'll make the robot drive from here (start point), and then, it will drive around there (end point), past the bag, and in like that.*
> Jacob: *Yes. How many times?*
> Lucas: *One. Until now.*

Both of the students and the teacher observed the robot while it was driving. The robot almost drove in a circle by ending almost at the starting place. Everybody got excited that the robot almost drove a circle. The teacher sat next to the students facing the computer and pointed to the screen:

> *Can you make a circle with only one such (program) so that it (the robot) drives around one circle.*
> Lucas: *I think so.*
> Jacob: *Make your circle a bit more specific. It might be like turning that way.*
> Lucas laughed enthusiastically and drew a circle with his hands: *Should it drive like that? Did you mean the pi-circle?*
> The teacher raised his shoulders and suggested: *Radius of 1 m, for example.*

Analysis based on Citation 3

The start of Session 2 was not smooth. The students had difficulties in collaborating, and it took quite a long time before the teacher came to see them. When the teacher came to see the students and realized that they were not working with the touch sensors anymore, he seemed relieved. After realizing that the students did not successfully complete the previous project, he took on a different kind of role and negotiated a new object with the students by suggesting one that was related to the situation they were engaged in. Students' questions and enthusiastic laughter indicated that they were interested in the teachers' suggestion.

Analysis based on Citations 1, 2, and 3

The comparison of Sessions 1 and 2 confirms the indication that the teacher had difficulties with the students' object during Session 1. This is manifested in a few different ways. First, the teacher did not come to see the students at the end of Session 1 after leaving. This is confirmed by the fact that it was a long time before the teacher came to see the students in the beginning because he thought that the students were still working on the previous project. Second, when the teacher came to see the students during Session 2, he immediately mentioned the object from the last time, and he laughed, relieved, on realizing that the students were not working on that object anymore. Third, he assumed a different role during Session 2 by discussing the assignment with the students in a different way. He used time for the students in a different way by sitting down and planning with them. He participated in the students' discussion, unlike during Session 1.

5.2. The role of the teacher – tools

The tools applied are based on the objects of the activity (Engeström, 1987):

Analysis based on Citations 1 and 2

During Session 1, programming tools were needed to mediate activity toward the object, which, in this case was the use of touch sensors to control the robot (Engeström, 1987). However, without sufficient tools, the students were not able to achieve their object and would have needed help and advice in programming. Their plan could not be implemented as such; however, the idea could have been developed with the help of the teacher. As also in Lindh and Holgersson (2007), the teacher was needed as technological support during students' learning processes with robots. However, the students did not get technological support from the teacher. As mentioned earlier, the teacher did not have the tools required to support the students. The students did not ask or even expect to get help from the teacher. They explained their plan and problems to the teacher when the teacher asked what they were doing. When they did not get any answer, they just continued working and did not mind that the teacher left.

Analysis based on Citation 3 and other field notes from Session 2

Session 2 had a view contrary to what Session 1 had regarding the use of tools. Apart from the programming tools, the students used mathematical tools that were appropriate for the object of the activity during Session 2. The teacher managed to mathematize the object by suggesting that the students could program the robot to drive a circle with a 1-m radius. The mathematized object needed mathematical tools to mediate the activity (Engeström, 1987). The students needed the circle perimeter formula to define the length of the route that the robot needed to drive. They also needed proportions to define how much the robot had to turn. The teacher did not intervene in the selection or use of these tools; however, the use of the tools was dependent upon the object, which, in turn, was the result of negotiation with the teacher.

The students did not have any difficulties with the use of programming tools during Session 2. The teacher saw that the students were able to program the robot to almost drive a circle. Thus, the teacher knew that the students had the programming tools required to achieve their object. The teacher's suggestion to have the robot drive in a circle was not challenging to their programming skills. It was rather challenging in its mathematical aspect, where the teacher had the tools required to guide the students.

During Session 1, the focus was on technological tools, and during Session 2, it was on mathematical tools, and the role of the teacher varied accordingly. The teacher was much more involved during Session 2 when the focus was on mathematical tools. He was able to understand and guide students during their entire learning process.

5.3. *The role of the teacher – collaboration among students*

At the beginning of Session 1, the students collaborated well and did not mind that they did not get any help from the teacher. However, the collaboration between the students collapsed after some time:

Citation 4 from our field notes (Session 1):

> Oscar worked relentlessly with programming and reasoning: *So, we must have this, at least. And then we should not use it, because when you press it then it stops just what it's connected to. The exclamation point… Of course, we have to connect something from here to them…I can do that. No. Yes… Listen. Come. Yes. We've got something, at least. So that we have to use… And then we have one, and then it's when the buttons are held in, something happens.*

The other students were no longer excited about the project. They did not listen to Oscar's reasoning. Oscar tried to get them to listen and got Lucas' attention.

> Oscar continued: *Yes, or it does not work. But, then we have to take up something, but what is the same?*

Lucas listened, but then, he continued building the Lego blocks. Jacob looked at what the other groups were doing and was worried about the time.

> Jacob: *How long have we got? Oh, ***, we have too little time. We have 17 min on us.*

Oscar continued his reasoning, and Lucas continued to build the Lego blocks; Jacob tried to listen to Oscar; however, he could not follow what Oscar was doing. He also reminded Oscar of the time.

Analysis based on Citation 4

The students became frustrated when the activity did not develop as desired, and the students' difficulties with collaboration escalated into a contradiction between subject and object in the activity system (Engeström, 2005). Because the activity did not develop in the desired way, the students developed different objects. Lucas and Jacob were no longer listening to Oscar. Oscar's reasoning was not consistent and became difficult to follow. Lucas started to build using Lego blocks, and Jacob was stressed with the time.

Analysis based on Citations 2 and 3

The students struggled to find a common object at the beginning of Session 2. Jacob had difficulties in following what Lucas was doing; however, the teacher's question regarding what students were doing got them to interact with each other. Jacob got a clue about what Lucas was doing, and both of them concentrated on the robot again. The students' new collective object, to program the robot to drive a circle with a 1-m radius, got students to collaborate again, and the activity got a new direction.

From this interaction, we can see that the role of the teacher can impact collaboration between students. The teacher's guiding questions were key to bridging the gap in understanding between the students. The situation in which Lucas and Jacob ended up with different objects because they had difficulty in following Oscar's reasoning might have been avoided with the help of the teacher's guiding questions. The example of this study supports the argument of Lindh and Holgersson (2007) that the teacher is needed to resolve conflicts among students during problem-solving activities with robots.

5.4. *Summary of the analysis*

The analysis above indicates that the role of the teacher differed during Sessions 1 and 2. The role of the teacher influenced object development, the tools in use, and collaboration among students during Sessions 1 and 2. These findings are summarized in Table 3.

As shown in Table 3, the development of the activity was based on object negotiation at the beginning of the sessions. It can be deduced from the teacher's role during Sessions 1 and 2 that the key to the development of the whole activity was the teacher's role during object negotiation. Barak and Assal (2018) argued that the teacher is needed at the beginning of the sessions, which is a claim supported by the comparison between Sessions 1 and 2. On the basis of our study, it could be said that the role of teachers at the

Table 3
Summary of the findings regarding the relationships between the role of the teacher and object development, the tools in use, and collaboration among students.

	The role of the teacher- object	The role of the teacher- tools	The role of the teacher- collaboration
Session1	The teacher was not present in the design phase of this session when students negotiated their object. This had an influence on activity development, such as tools in use and collaboration among students.	Students did not have the tools required to obtain their object. This could have been avoided if the teacher was involved earlier in object negotiation. The teacher did not have the tools required to guide the students in this phase.	At the end of the session, students' collaboration was not successful. This was a consequence of activity development that was based on the object of the activity and tools in use. This could have been avoided had the teacher been involved in the previous phases with some of his questions as a guide.
Session2	By making suggestions on the basis of students' original activity, the teacher and the students negotiated a common object together. This had an influence on the development of activity, such as tools in use and collaboration among the students.	Students had the programming tools required to obtain their object. In addition, mathematical tools were in use. The tools applied were based on the object negotiated with the teacher. Moreover, the teacher had the tools needed to guide the students, such as his mathematical and pedagogical knowledge.	Students had difficulties with collaborating among themselves in the beginning. This situation was solved by the teacher's guiding questions and negotiation at the beginning of the session. After the collective object was negotiated, collaboration among the students was successful during the development of the whole activity. This was based on the object of the activity and tools in use.

beginning of the activity is key to the development of the whole activity. First, this is because teachers are able to help the students during their learning processes by utilizing their (teachers') acquired mathematical and pedagogical knowledge. On the basis of studies conducted using the TPACK model, the manner in which programming can be integrated into mathematics education depends on teachers' technological knowledge (Guerrero, 2010). We argue that teachers can compensate for their lack of programming skills with their pedagogical knowledge and by using object negotiation with the students.

Second, teachers are capable of indirectly influencing the use of different tools during the learning process through the object. The reason is that students had mathematical tools in use, and they managed with their programming tools in use.

Third, the role of teachers affects students' collaboration. According to previously conducted studies, technology, programming, and robot integration support collaborative learning in mathematics classrooms (Forsström & Kaufmann, 2018; Bray & Tangney, 2017; Martinovic et al., 2013). We proved that collaboration among students depends on the teacher's role, particularly in the object negotiation phase.

6. Discussion

Even though the sessions observed and analyzed for this article were not part of a mathematics class, the observations are transferable to the mathematics classroom because the activities support the mathematics curriculum with the development of problem-solving skills and the use of mathematical tools. We observed a circumstance under which it was possible to integrate programming into mathematics even when the mathematics teacher did not have a technological background.

The key difference between the data from the two sessions that we observed was the role of the teacher. The task during Session 1 was highly student-centered, and the teacher did not participate in the activity. The students were the only subjects in the activity, and activity development was dependent on the tools that they were using. The availability of tools to the subjects depended on their knowledge and histories (Engeström, 2005). However, they did not have the required knowledge; hence, they were not able to make progress.

During Session 2, the teacher renegotiated the object of the activity together with the students. Thus, the teacher was one of the subjects, together with the students. The teacher influenced object development with his knowledge and tools. The object eventually agreed upon by the students and the teacher kept the students and the teacher motivated throughout the activity.

These sessions show that the key to the successful use of robots appears to be fruitful teacher intervention for the negotiation of objects that are obtainable by the students and pedagogically useful. In this case, the teacher and the students worked together as subjects of the activity toward the same object, and thus, they have the knowledge and tools of both the teacher and the students. When they worked together, their knowledge was stronger and they had more tools to use, which enabled fruitful activity development.

The model that we used could also be used to analyze the integration of technology into a mathematics classroom where the teacher is not an expert in technology. According to earlier studies, although the integration of technology has the potential to change the culture of traditional mathematics classrooms to be more student-centered (Bray & Tangney, 2017), teachers without strong technological knowledge still tend to integrate technology using a teacher-led approach (e.g., Guerrero, 2010; McCulloch et al., 2018). In a traditional, teacher-led classroom, the students and the teacher have separate activity systems, and each uses their own tools to achieve their individual objects. Traditionally, students are objects in the teacher's activity system, which we typically call "teaching." The learning activities of students have a different object, which is frequently the completion of a given task (Engeström, 2008; Martinovic et al., 2013). In a totally student-centered approach, the students have their own separate activity system, and they use their own tools to achieve the objects that they themselves created. In such a system, they are on their own if the teacher is not able to help them. Even if they have good technological knowledge, the teacher's guidance is needed if technology is to be successfully integrated into the classroom. The mathematics teacher often has strong pedagogical and mathematical knowledge on the basis of their history as a teacher even if they lack technological knowledge. Neither the students nor the teacher knows everything, and one key is that students and the teacher know different things. The use of technology in the classroom is most effective when the teacher and students pursue a common object together rather than pursuing two disparate objects. Through the common object, the teacher brings their knowledge and tools in addition to the students' knowledge and tools to the activity that the students are engaged in.

This article addressed the very beginning of programming integration. This paper might not have a report on what happened over time when the students and the teacher were more familiar with the programming tools; however, it gave valuable information from the beginning of programming integration on situations with the potential to further development. However, it was obvious at the beginning of the sessions that the development of the whole activity depended on the teacher's role as object negotiator, which provided a good basis for future activities.

Another question of interest is the following: what happens when students want to develop their innovative ideas and need technological advice? We proved that the teacher's role as an object negotiator provides a good basis for this because it creates a conducive, facilitative environment for the teacher and the students to work together and use their knowledge to solve emerging problems. Moreover, there is a need for many more studies to be conducted on the questions raised in this paper and others. The observations made during this study support the idea that there is a need for further education and peer support for teachers who do not have previous knowledge regarding programming but wish to or must integrate programming into their classrooms (e.g., Balanskat & Engelhardt, 2015; Bocconi et al., 2018).

7. Conclusions

This article has explored the impact of the role of the teacher in the integration of robots into the mathematics curriculum on students' mathematics learning processes. Our findings suggest that the choices of the teacher in the activity design phase are the most essential element supporting students' whole learning process with robots. It enables fruitful mathematics when the teacher is an active and engaging object negotiator at the beginning of the sessions in technology integration. These learning processes depend on the interactions between the teacher and students, and therefore, these cannot be predicted or controlled in advance (Engeström & Sannino, 2012).

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